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Statistical Analysis and Variability of Pavement Materials

by Reed B. Freeman, William P. Grogan

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Waterways Experiment Station
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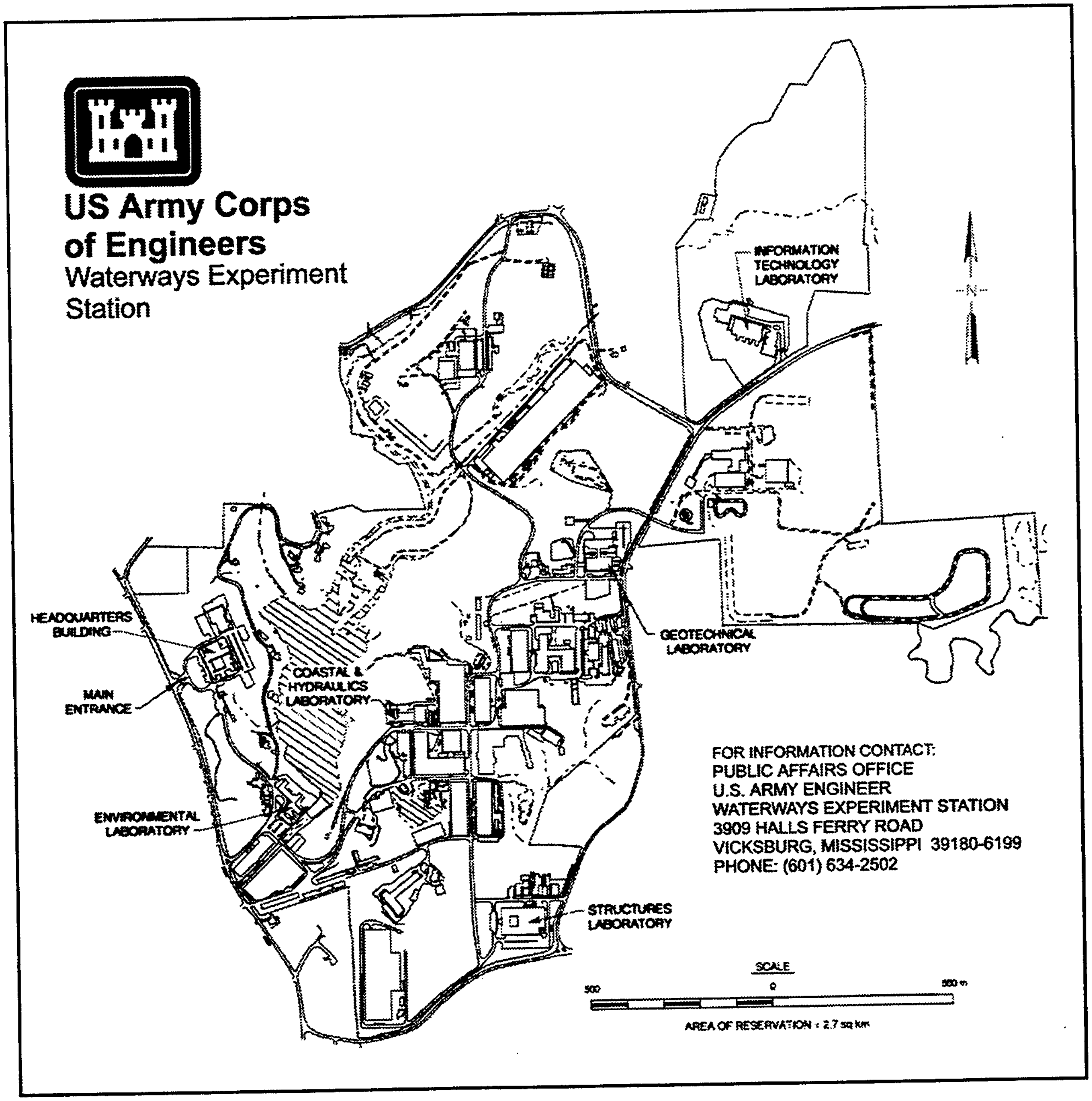

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Preface

The investigation documented in this report was sponsored by the U.S. Army Corps of Engineers through the Research, Development, Testing, and Evaluation (RDT&E) Program, Pavement Technology Work Package, Work Unit AT40-PT-014, "Characterization and Variability of Pavement Materials." The Corps of Engineers Technical Monitor was Mr. Ray Navidi, CEMP-ET.

This research was conducted by personnel of the Airfields and Pavements Division (APD), Geotechnical Laboratory (GL), at the U.S. Army Engineer Waterways Experiment Station (WES), Vicksburg, MS.

This study was conducted under the general supervision of Dr. William F. Marcuson III, Director, GL. Direct supervision was provided by Dr. Ray S. Rollings, Acting Chief, APD, and Mr. Timothy W. Vollor, Chief, Materials Analysis Branch (MAB), APD. The principal investigator for the project was Dr. Reed B. Freeman, MAB. The report was authored by Dr. Freeman and Mr. Bill Grogan, MAB.

Director of WES during the conduct of this study and preparation of the report was Dr. Robert W. Whalin. The Commander was COL Bruce K. Howard, EN.

1 Introduction

For the past several decades, the pavement construction industry has moved away from material and method specifications, toward quality assurance specifications. Material and method specifications required complete direction of construction by a representative of the contractual "owner." The contractor was directed to use specific materials and specific methods of placement. This type of specification created several problems. It prevented the contractor from using innovative materials or techniques and it obligated the owner to accept the final product regardless of quality.

Quality assurance specifications depend primarily on end-result criteria, which require the contractor to take responsibility for supplying a product or an item of construction. The owner's responsibility is then either to accept or reject the final product or to apply a price adjustment that compensates for the degree of compliance with the specifications (TRB 1996).

The end-result criteria used for a quality assurance specification must be related to pavement performance. These criteria involve testing for material and structural characteristics that are known to correlate with pavement performance (e.g. void content and thickness for asphalt concrete). The characteristics used for end-result criteria are often not fundamental engineering properties (e.g. resilient modulus or fatigue resistance) because they have to be highly repeatable and amenable to timely acceptance testing (TRB 1996).

To execute a quality assurance specification properly, the contractor must implement a quality control (QC) program and the owner must implement a quality assurance (QA) program. A QC program involves sampling and testing during construction to control the level of quality of the final product. A QA program involves all the actions necessary to provide confidence that a product or facility will perform satisfactorily in service. The QA program must include acceptance sampling and testing to determine if the quality of produced material or construction is acceptable in terms of the specifications (TRB 1996).

The results of QA acceptance tests are compared to acceptance limits, which usually include provisions for pay adjustments. These acceptance limits must be realistic, particulary when nonconformance results in reduced pay for a contractor. The principal obstacle for developing realistic limits is

the ability to predict with some degree of accuracy the amount of variability that should be expected in "satisfactory" construction (Nicotera 1974).

Problem

The pavement industry has not maintained or published extensive variability data related to materials and construction, as have other fields of engineering and science. Pavement construction has long been regarded as an art that is highly dependent on the skills and experience of the field engineer, so decisions have often been based on engineering judgement. The use of traditional material and method specifications for pavement projects has been accompanied by a minimal amount of conformance testing. Consequently, there has typically not been an organized method for resolving disputes when materials or methods used for pavement construction are suspected to not conform with specification requirements. Other industries, in contrast, involve production lines in a controlled environment that includes conformance testing. Successful automobile manufacturing, for example, has relied on a statistical approach to quality control for over 40 years.

The pavement industry is now adopting statistics-based specifications and reliability assessments for several reasons. First, the pace of pavement construction projects has increased, necessitating that the parties involved share responsibilities. Secondly, the duties of engineers has become more widespread, requiring that they spend time away from their field projects. Finally, modern construction has been besieged with litigation. Owners must therefore protect themselves by writing clear specification requirements that are based on fair assumed risks by all contractual parties.

Scope

This report has two purposes. First of all, it describes probabilistic and statistical tools that are useful for analyzing the variability of pavement materials and pavement structures. Secondly, this report presents a summary of published variability data pertaining to pavements. These tools and data will be useful for the development of fair and enforceable quality assurance specifications.

Basic statistical concepts are reviewed in Chapter 2 to prepare the reader for the variability analysis tools presented in Chapter 3. Published variability data are summarized in Chapter 4 and are presented in detail, with references, in the appendices.

2 Statistical Concepts

This chapter provides a review of basic probabilistic and statistical concepts, which will be used for analyses of material variability later in this report. This chapter should be useful to those who develop, write, and implement statistics-based construction specifications.

Terminology

Population versus sample

A population is a set of data that includes an entire entity of interest. If an Army installation is conducting inventory on its highway system, the design thicknesses of all portland cement concrete (PCC) pavements would be considered a population of data. A sample is a set of data that is selected randomly from within a population, typically to represent the population from which it was derived. If an Army installation were to make a judgement concerning the thicknesses of its PCC pavements by coring 100 randomly-selected slabs, the selected slabs would constitute a sample of the installation's population of PCC pavements.

Unit, observation, and variable

A set of data is composed of information gathered from a number of experimental units. The information gathered from each experimental unit is called an observation. Each observation may consist of one or more pieces of information, called variables.

Example. If an Army installation wishes to gather information about nearby sources of asphalt cement, each source of asphalt cement would be a unit. The observation for each of these units may consist of several variables, such as specific gravity, solubility, and viscosity at a given temperature.

Types of variables

Variables may be continuous or discrete. Continuous variables can take on any value within a given interval.

Example. The asphalt cement content of hot-mix asphalt (HMA), which could be measured from extractions performed on pavement core samples, is a continuous variable. Conceptually, the asphalt cement content could be any value between and including 0 percent and 100 percent. Similar to almost all continuous variables, the reported measurements of asphalt cement content are limited to discrete increments due to the limited precisions of measurement devices. Reported asphalt cement contents are typically limited to increments of one-tenth of a percent.

A discrete variable must assume a single category from within a predetermined list. These categories may be based on a nominal scale or an ordinal scale. In a nominal scale, each category is given a name. The category names may be alphabetic, numeric, or alphanumeric.

Example. If an Army installation is conducting inventory on its roadway system, they may classify each pavement section by an alphabetic name that reflects the nature of its design: rigid, flexible, or unsurfaced. As an example of numeric categories in a nominal scale, an installation may classify its roadways according to their number of lanes. All observations for the number of lanes would be discrete; commonly, 2 or 4. In this case, the discrete variable names would have quantitative implications.

An ordinal scale for a discrete variable involves a ranking procedure. These ranks are typically given integer values including "1" for either the lowest rank or the highest rank.

Example. If an Army installation is concerned with surface scaling deterioration for reinforced concrete bridge decks, they may inspect their bridge decks visually and then rank the appearances of the bridge decks with an ordinal scale.

Frequency Distributions

A frequency distribution is a fundamental method for summarizing data. In order to construct this distribution, descriptive categories (or class intervals) are established so that each data observation falls into a single category. The number of observations that fall within each category are then counted and tabulated. The necessary descriptive categories may exist naturally for discrete variables, but they must be fabricated for continuous variables. The appropriate number of class intervals is usually between 5 and 20, depending on the number of observations and the apparent trends in the data (Spiegel 1988). The frequency of occurrences for the various class intervals form the basis of a frequency distribution.

Example. Frequency of occurrences for a continuous variable, California bearing ratio of a silty gravel, are developed in Table 2-1.

Table 2-1
California Bearing Ratio for a Silty Gravel

Class Interval	Class Mark[1]	Frequency	Relative Frequency	Cumulative Relative Frequency
25 to 29	27	2	0.018	0.018
30 to 34	32	4	0.036	0.055
35 to 39	37	7	0.064	0.118
40 to 44	42	11	0.100	0.218
45 to 49	47	15	0.136	0.354
50 to 54	52	18	0.164	0.518
55 to 59	57	20	0.182	0.700
60 to 64	62	14	0.127	0.827
65 to 69	67	12	0.109	0.936
70 to 74	72	5	0.046	0.982
75 to 79	77	2	0.018	1.000
Total		110	1.000	

[1] Midpoint of class interval.

Once the frequencies of occurrence are established, they can be converted to relative frequencies by expressing the number of observations for each category as a proportion of the total number of observations (Table 2-1). Each relative frequency, which takes a value between 0.0 and 1.0, represents an estimated probability that an observation will take on a value within its descriptive data category. For this reason, relative frequencies are often referred to as empirical probabilities (Freund and Wilson 1993). Once relative frequencies are established, they can be converted to cumulative relative frequencies, which also take values between 0.0 and 1.0 (Table 2-1). Cumulative relative frequencies are often referred to as empirical cumulative probabilities.

A histogram is a graphical representation of either frequencies of occurrence or relative frequencies (Figure 2-1). These histograms are often referred to as frequency distributions or relative frequency distributions, depending on the type of ordinate used. These histograms provide an easy-to-interpret view of both the range of measurements and the shape of the distribution of measurements. Important characteristics of the shape include symmetry and data dispersion. Overlaying the histogram with a cumulative

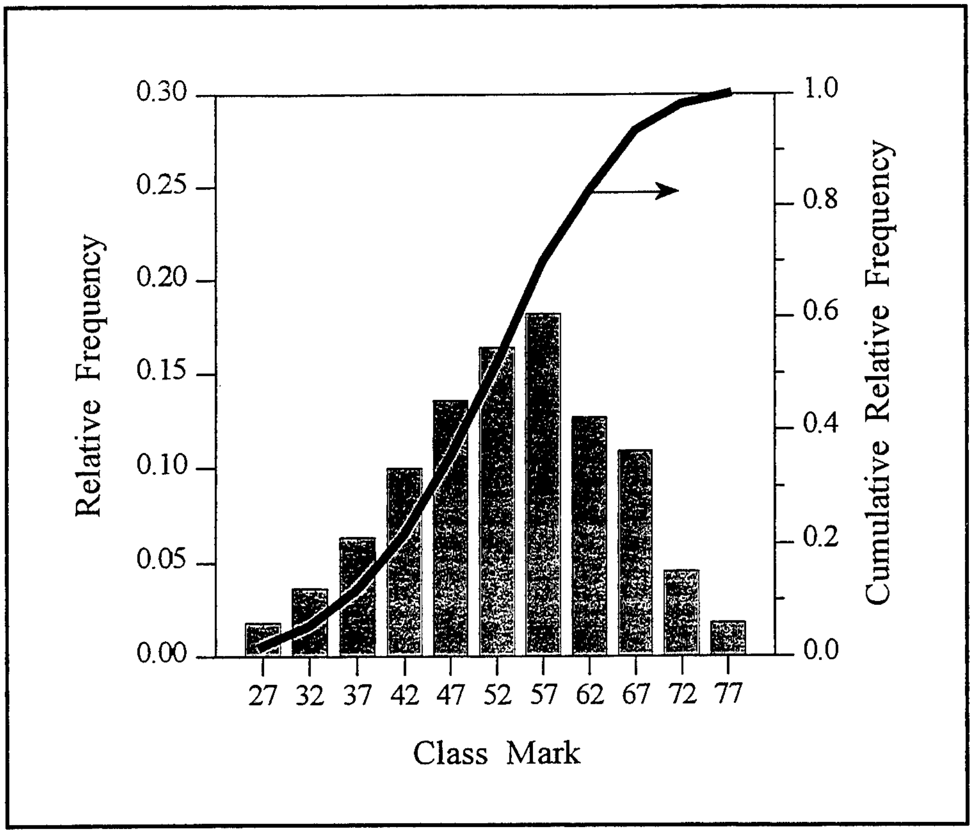

Figure 2-1. Relative frequency distribution for soil CBR

relative frequency plot provides the viewer with an additional perspective of the same data, as shown in Figure 2-1.

Numerical Descriptive Statistics

Although frequency distributions provide useful descriptions of data, numerical descriptors are often needed for quantitative comparisons between data sets. When numerical descriptors are used to represent frequency distributions, however, some information may be lost (e.g. the shape of the frequency distribution). For this reason, numerical descriptors must be used with care and with knowledge of any accompanying assumptions.

Central tendency

One of the most useful single characteristics of a distribution is its central tendency. Central tendency can be calculated several ways, including those listed below (Freund and Wilson 1993). These methods of calculating central tendency are valid for both populations of data and samples of data. Mean is the most commonly used measure of central tendency.

a. Mean. The mean is the sum of all values (assuming a finite population size) divided by the number of values. A mean for a population is commonly denoted μ and a mean for a sample is commonly denoted $\bar{y}$ (where the individual sample observations are denoted y_i). As an example, the mean of the relative density data in Table 2-2 is 97.33 (=1557.2/16).

Table 2-2
Relative Densities for Hot Mix Asphalt

Ordered Observation	Ordered Measurement (Percent)	$y_i - \bar{y}$[a]	$(y_i - \bar{y})^2$	$(y_i - \bar{y})^3$
6	96.8	-0.53	0.276	-0.145
7	96.9	-0.42	0.181	-0.077
15	97.0	-0.33	0.106	-0.034
5	97.1	-0.23	0.051	-0.011
13	97.2	-0.13	0.016	-0.002
1	97.3	-0.03	0.001	0.000
4	97.3	-0.03	0.001	0.000
9	97.4	0.08	0.006	0.000
14	97.4	0.08	0.006	0.000
2	97.5	0.17	0.031	0.005
10	97.5	0.17	0.031	0.005
11	97.5	0.17	0.031	0.005
16	97.5	0.17	0.031	0.005
3	97.6	0.27	0.076	0.021
8	97.6	0.27	0.076	0.021
12	97.6	0.27	0.076	0.021
sum	1557.2	0.00	0.99	-0.18

[a] $\bar{y}$ = 97.33 percent.
Note: Relative density = (field density/lab density) x 100 percent.

b. Median. The median is the middle value when the measurements are arranged from lowest to highest. The median will be the average of two values if the number of measurements is even. The median of the relative density data in Table 2-2 is 97.4.

c. Mode. The mode is the most frequently occurring measurement. The mode will not be a unique value if two or more measurements occur with the same greatest frequency. The mode will not be defined if each measurement occurs only once. The mode of the relative density data in Table 2-2 is 97.5.

d. Geometric Mean. The geometric mean is the N[th] root of the product of N values. The geometric mean will not be defined if any measurements are less than or equal to zero. The geometric mean of the relative density data in Table 2-2 is 97.31.

e. Midrange. The midrange is the average of the smallest and largest measured values. The midrange is not often used because it ignores most of the information provided by the data. The midrange of the relative density data in Table 2-2 is 97.20.

Dispersion

The variability or dispersion of a data set is also important and should be quantified if possible. A value for dispersion will provide an indication of whether a frequency distribution is "broad" or "narrow," however, it will not provide an indication of symmetry (or skewness). The simplest measure of dispersion is the range, which is defined as the difference between the largest and smallest observed values. Similar to the calculation for mid-range, the calculation for range ignores most of the values in a data set and therefore has limited usefulness.

Another measure of dispersion, called variance, is commonly used. Variance is represented by two symbols, depending on its application (Steel and Torrie 1980): σ^2 is used if the data set is a population and s^2 is used if the data set is a sample. The population variance (assuming discrete observations) is defined as the sum of squared deviations from the mean, divided by the total number of observations, N:

$$\sigma^2 = \frac{\sum_{i=1}^{N} (y_i - \mu)^2}{N} \tag{1}$$

where

y_i = value for each observation

μ = population mean

N = number of units in the population

The numerator in Equation 1 is often referred to as the "sum of squared deviations" or the "sum of squares." In order to simplify computations, the sum of squares equation can be transformed to a working formula:

$$\sum_{i=1}^{N} (y_i - \mu)^2 = \sum_{i=1}^{N} y_i^2 - \frac{\left(\sum_{i=1}^{N} y_i\right)^2}{N} \tag{2}$$

The calculation of sample variance, s^2, is similar to the calculation of population variance with the exception that the divisor is $(n-1)$, rather than N:

$$s^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1} \tag{3}$$

where

y_i = value for each observation

$\bar{y}$ = sample mean

n = number of units in the sample

The necessity of using $(n-1)$ as the divisor for the sample variance (Equation 3), rather than n, is a consequence of using a sample statistic, $\bar{y}$, as an estimate of a population parameter, μ, in its calculation. While population parameters are fixed values, sample statistics are random variables. This idea will be expanded upon in the section titled, "Sampling Distributions."

For the relative density data in Table 2-2, variance is calculated as:

$$s^2 = \frac{0.99}{15} = 0.066\ percent^2 \tag{4}$$

In addition to range and variance, the dispersion of a data set may be quantified by its standard deviation. Standard deviation is simply the square root of the variance, as shown below. At times, the standard deviation is convenient because its units are the same as the units for the data from which it is calculated.

$$\sigma = \sqrt{\sigma^2} \qquad and \tag{5}$$

$$s = \sqrt{s^2} \tag{6}$$

where

σ = standard deviation for a population

s = standard deviation for a sample

For the relative density data in Table 2-2, standard deviation is calculated as:

$$s = \sqrt{s^2} = \sqrt{0.066} = 0.257 \textit{ percent} \tag{7}$$

Since sample standard deviations and variances are themselves variables, several independent estimates for these statistics may be available. The proper method for combining these estimates is to calculate a weighted average for variance, with the weights proportional to the respective degrees of freedom [Granley 1969, ASTM (1995b) E 178]. The calculation of weighted average for variance is often called "pooling" the variance estimates:

$$\textit{pooled sample variance} = \frac{\sum_{i=1}^{M} [(s_i^2)(n_i - 1)]}{\sum_{i=1}^{M} n_i - M} \tag{8}$$

where

n_i = number of tests in project I

M = number of projects

Pooling is essentially a weighted average with the degrees of freedom in each project (n_i-1) as the weighting factor. If the number of tests within each project are the same, the pooled value is equal to the straight average.

Finally, the dispersion of a data set can be quantified by a coefficient of variation (CV), as shown below. The coefficient of variation can be described as the standard deviation expressed as a percentage of the mean. This measure of dispersion has advantages at times because the magnitude of the mean and the units of measure are factored out. Using CV, distributions representing data with different units and distributions representing data with different means can all be compared in terms of their dispersion. Equations for computing CV for both populations and samples are shown below.

$$CV = \frac{\sigma}{\mu} \times 100\% \qquad \textit{or} \tag{9}$$

$$CV = \frac{s}{\bar{y}} \times 100\% \tag{10}$$

For the relative density data in Table 2-2, coefficient of variation is calculated as:

$$CV = \frac{0.257}{97.33} \times 100\% = 0.26\% \tag{11}$$

Shape

Skewness is a measure of the degree of symmetry for a frequency distribution. If the right tail of a frequency distribution extends farther from the central maximum than the left tail, the distribution is said to be skewed positively (Figure 2-2). If the left tail of a frequency distribution extends farther from the central maximum than the right tail, the distribution is said to be skewed negatively (Spiegel 1990). The moment coefficient of skewness for a population (β_1) is calculated as:

$$\beta_1 = \frac{\sum_{i=1}^{N} (y_i - \mu)^3}{N \cdot s^3} \tag{12}$$

The moment coefficient of skewness for a sample (b_1) is calculated as (SAS 1988):

$$b_1 = \frac{n}{(n-1)(n-2)} \times \frac{\sum_{i=1}^{n} (y_i - \bar{y})^3}{s^3} \tag{13}$$

If a distribution is positively skewed, its moment coefficient of skewness will be positive. If a distribution is negatively skewed, its moment coefficient of skewness will be negative. If a distribution is symmetric, its moment coefficient of skewness will be equal to zero. By raising the standard deviation in the denominator to a power of three, the moment coefficient of skewness becomes unitless.

For the relative density data in Table 2-2, coefficient of skew is calculated as:

$$b_1 = \frac{16}{15 \cdot 14} \times \frac{-0.18}{(0.257)^3} = -0.83 \tag{14}$$

Discrete Probability Distributions

Relative frequency distributions were described previously as empirical probability distributions. Relative frequency distributions are considered empirical because they are usually constructed from sample values. Relative frequency distributions can also be called "discrete" distributions because they are constructed from data that has been categorized into discrete class intervals.

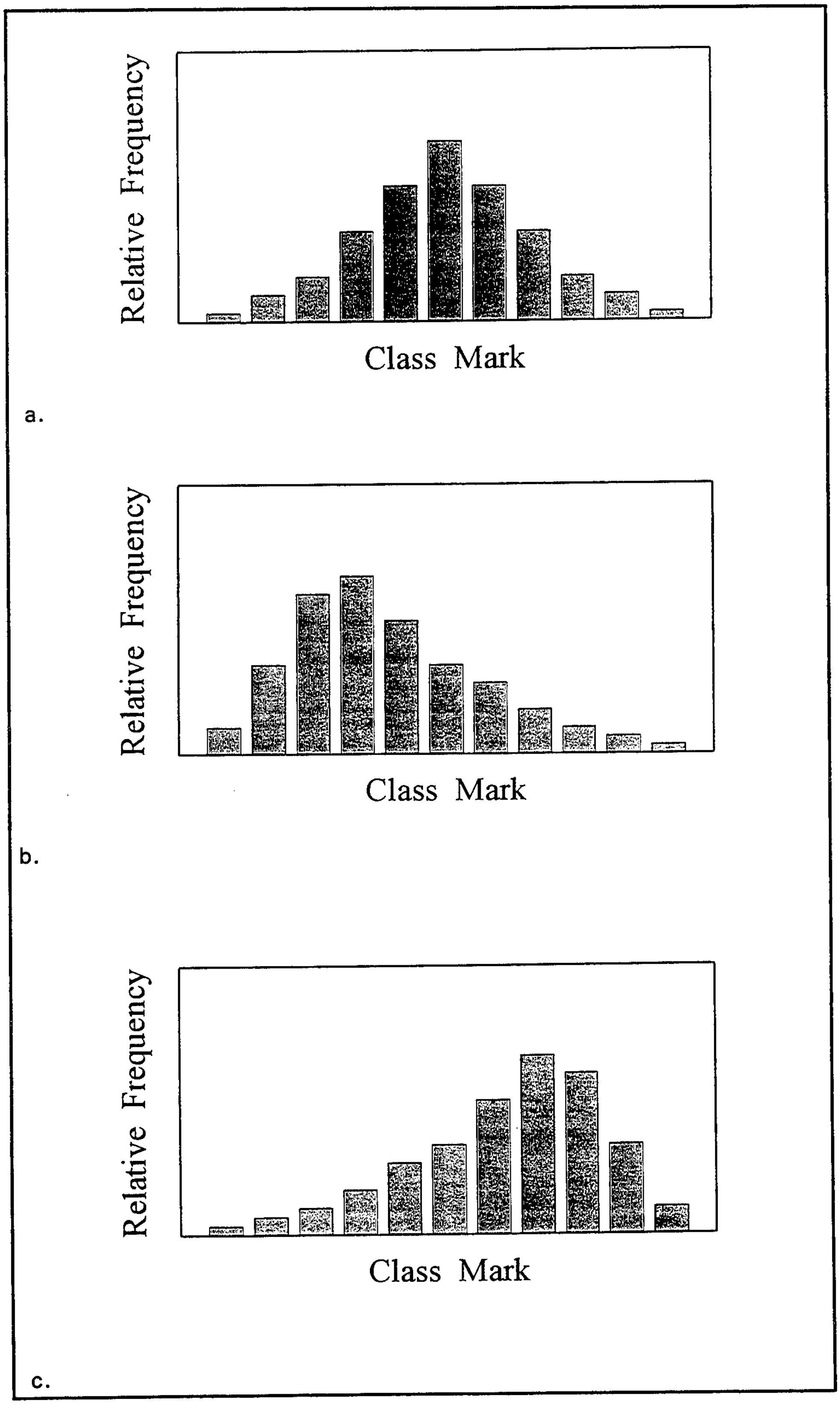

Figure 2-2. Relative frequency distributions with various skew: (a) symmetric, (b) positive skew, and (c) negative skew

Contrary to relative frequency distributions, which are constructed from sample data, the construction of probability distributions is based on assumptions related to either the shape of the distribution or the probability of occurrence for each discrete value. Probability distributions are therefore theoretical, rather than empirical, and represent theoretical characteristics of a population of data.

Binomial distribution

A relatively simple and useful discrete probability distribution is the binomial distribution. This distribution is applicable for statistical problems that include "repeated trials." In these situations, we are interested in the probability of getting x successes in n trials. In order for the binomial distribution to apply, the following assumptions must also be true (Johnson 1994).

a. There are only two possible outcomes for each trial (usually referred to as "success" and "failure").

b. The probability of success (p) is the same for each trial.

c. The n trials are independent.

Let Y be the random variable that represents a set of all possible successes in n trials. The probability of the occurrence of y successes, which is represented as $p(y)$, can be calculated for each possible value of y. The binomial distribution can then be constructed by plotting $p(y)$ as a function of the discrete values of y. In order to emphasize the fact that $p(y)$ is calculated for a binomial distribution, $p(y)$ is often denoted by $b(y; n, p)$.

$$b(y;\, n,\, p) = \binom{n}{y} p^{y}(1-p)^{n-y} \; for \; y = 0,1,2,\ldots,n \qquad (15)$$

where

$$\binom{n}{y} = \frac{n!}{y!\,(n-y)!} \qquad (16)$$

Cumulative probabilities of the binomial distribution are often needed for statistical applications. The cumulative binomial probability, B, can be calculated as follows:

$$B(k;\, n,\, p) = \sum_{y=0}^{k} b(y;\, n,\, p) \; for \; k = 0,1,2,\ldots,n \qquad (17)$$

The mean and the variance for the binomial random variable y can be calculated without constructing the probability distribution, provided that the two distribution parameters n and p are known.

$$\mu = np \qquad and \tag{18}$$

$$\sigma^2 = np(1-p) \tag{19}$$

If n is large, calculating the binomial probabilities and the cumulative binomial probabilities can be cumbersome. Statistical software packages are often equipped to handle these calculations. In addition, many textbooks include an appendix with cumulative binomial probabilities tabulated for a range of values for each of y, n, and p.

Example. Assume that a contractor is removing concrete cores from a rigid pavement. The contractor needs four cores that remain intact and are of sufficient length for compression testing (successes). He/she knows from experience that only about one-half of the cores attempted are of sufficient quality (successes), so $p=0.5$. If the contractor's crew is cutting 12 cores ($n=12$), his/her binomial distribution for this situation is shown in Figure 2-2. The probability that the contractor will obtain three or fewer quality cores is only 7 percent. The probability that the contractor will obtain the necessary four quality cores, or even more, is approximately 93 percent.

Continuous Probability Distributions

Probability distributions can also be constructed for continuous random variables, which can assume an infinite number of different values (any value within an interval). Continuous probability distributions for these continuous variables have the following characteristics (Freund and Wilson 1993).

a. The graph of the distribution is usually a smooth curve. This is in contrast to the histogram type of distribution for discrete variables.

b. The curve is described by an equation, $f(y)$, called the distribution function. This function corresponds to $p(y)$ for discrete variable distributions.

c. The total area under the curve is one. This corresponds to the sum of all possible probabilities for discrete variables.

d. The probability of a random variable taking a value within an interval (a, b) is equal to the area between the distribution curve and the horizontal axis within the interval (a, b). This corresponds to adding probabilities for discrete variables.

Normal distribution

The normal distribution is a continuous distribution that has proved to be useful in many applications. The normal distribution was developed in the eighteenth century when scientists observed regularity in errors of measurement, caused by laws of chance. They called the family of distributions the "normal curve of errors" and fit the following continuous equation, often referred to as the normal probability density function (Johnson 1994).

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right] \tag{20}$$

where

$$-\infty < y < \infty$$

The position and breadth of the resulting normal probability density function is dependent on only two parameters: mean (μ) and standard deviation (σ). The curve is always bell-shaped and symmetric about the mean, so its degree of skewness is always zero. The breadth of the curve increases with increasing σ. This effect is shown in Figure 2-3 for functions with a mean of 0 and standard deviations of 1/2, 1, and 2. While the breadth of normal distributions can change, the following rules of thumb remain applicable for data dispersion (Willenbrock 1974b).

a. Approximately 68.0 percent of the data falls within plus or minus one standard deviation from the mean ($\mu \pm \sigma$).

b. Approximately 95.5 percent of the data falls within plus or minus two standard deviations from the mean ($\mu \pm 2\sigma$).

c. More than 99.7 percent of the data falls within plus or minus three standard deviations from the mean ($\mu \pm 3\sigma$).

Standard normal distribution

The calculus required to integrate under normal distribution curves can be quite complex, so probabilities related to areas under these density functions are seldom calculated without the aid of computer software. If software is not convenient for a particular application, tables representing a standard normal distribution can be used to facilitate manual calculations. The standard normal distribution has a mean of zero and a standard deviation of one, resulting in the probability density function shown below.

$$f(y) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-y^2}{2}\right] \tag{21}$$

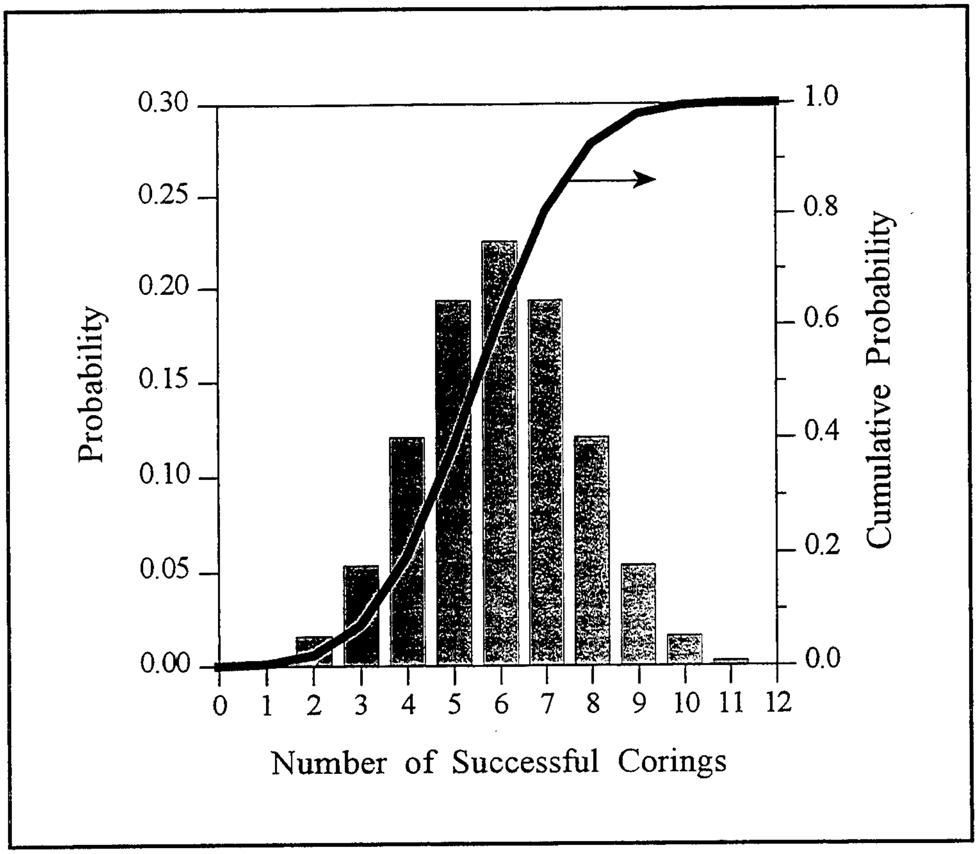

Figure 2-3. Discrete binomial distribution

where

$$-\infty < y < \infty$$

For all normal distributions, the degree of skewness is zero.

The random variable associated with the standard normal distribution is usually represented by the letter $Z\ [=\{z\}]$. Areas under a normal distribution for any variable $Y\ [=\{y\}]$ can be determined by converting a specific value y to a value z according to the equation below. The calculated value z can then be used as input for standard normal distribution tables, such as that shown in Table A1 (Appendix A), to determine the probability of finding measurements less than or greater than y.

$$z = \frac{y-\mu}{\sigma} \tag{22}$$

Example. Assume that a specific mixture of portland cement concrete is required to have a compressive strength greater than or equal to 34 MPa and

that the concrete supplier involved has historically reported coefficients of variation for strength in the range of 20 percent. If the average compressive strength for the delivered concrete is 40 MPa, we can use the "Z" tables to calculate the probability that a measured compressive strength will fall below the specified 34 MPa. From the given values for coefficient of variation and mean, the standard deviation of compressive strengths is calculated to be 8 MPa (0.2 x 40 MPa). The z-value that corresponds to a y-value of 34 MPa equals -0.75. Since the standard normal distribution is symmetric, we know that the probability of obtaining a value of z less than -0.75 is equal to the probability of obtaining a value of z greater than +0.75. Therefore, using the standard Z table (Table A1) and interpolation, we can determine that the probability of measuring a compressive strength smaller than the specified 34 MPa equals 0.2266 or 23 percent. The assumed normal distribution for compressive strength, along with a cumulative probability distribution, are plotted in Figure 2-4. As expected, the cumulative probability at 34 MPa is approximately 0.23.

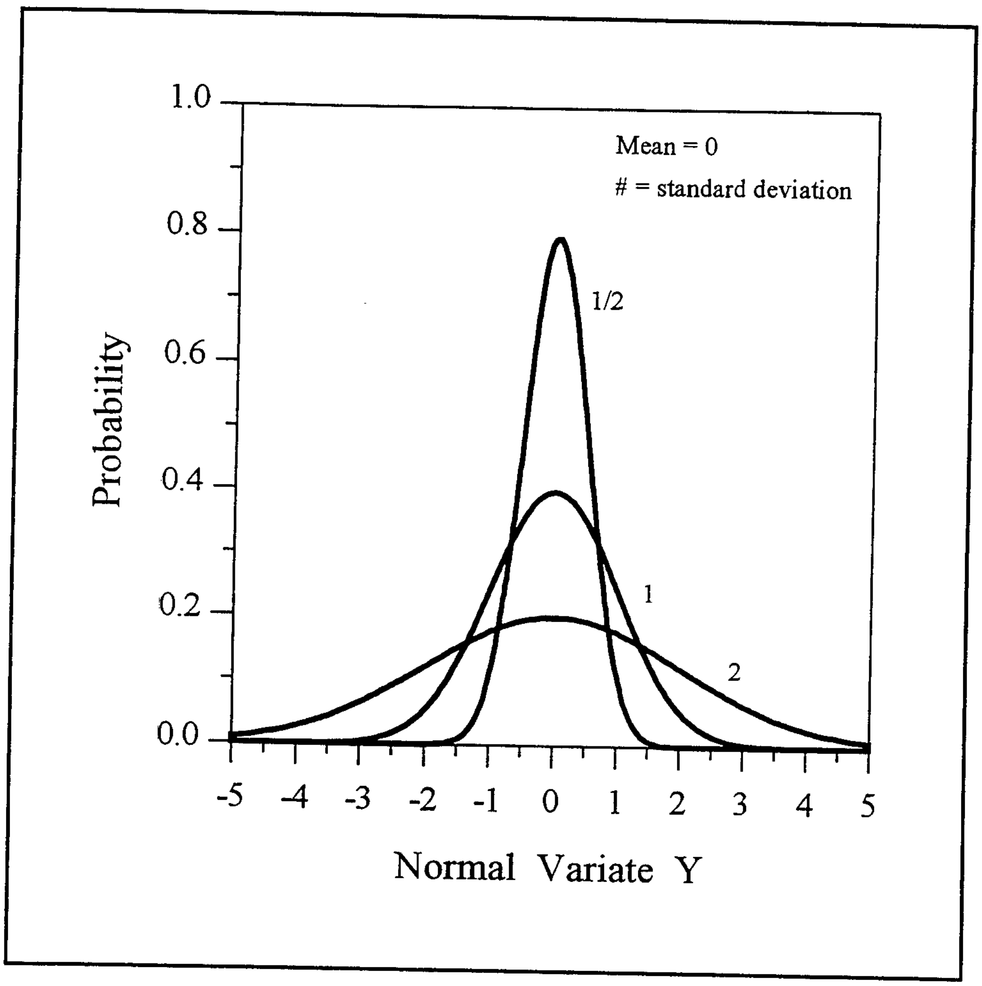

Figure 2-4. Effect of standard deviation on the breadth of a normal distribution

Beta distribution

The normal distribution is not the most appropriate representation for all applications. Some random variables are represented more accurately by distributions with finite limits. For example, California bearing ratio (CBR) measurements for a high-plasticity clay would have a mean close to zero and a relatively high standard deviation. However, the real-life measurements could never have a value less than zero. In cases such as this and for skewed distributions, a beta distribution may be more applicable than a normal distribution. Beta probability distributions are continuous and they have a finite range from y=a to y=b. These probability distributions can be described by the density function shown below (Harr 1987).

$$f(y) = C(y-a)^{\alpha}(b-y)^{\beta} \tag{23}$$

where

$$\alpha > -1,\ \beta > -1,\ \textit{and} \tag{24}$$

$$C = \frac{(\alpha+\beta+1)!}{\alpha!\,\beta!\,(b-a)^{\alpha+\beta+1}} \quad (\alpha \textit{ and } \beta \textit{ are integers}) \tag{25}$$

Given a, b, α, and β for a particular beta distribution, the mean and the variance of the distribution can be calculated as shown below.

$$\mu = a + \frac{\alpha+1}{\alpha+\beta+2}(b-a) \tag{26}$$

$$\sigma^2 = \frac{(b-a)^2(\alpha+1)(\beta+1)}{(\alpha+\beta+2)^2(\alpha+\beta+3)} \tag{27}$$

Given a range, a mean, and a standard deviation for a random variable Y, values of α and β for the appropriate beta distribution can be calculated as shown below.

$$\alpha = \frac{P^2}{Q^2}(1-P) - (1+P) \qquad \textit{and} \tag{28}$$

$$\beta = \frac{\alpha+1}{P} - (\alpha+2) \tag{29}$$

where

$$P = \frac{(\mu - a)}{(b - a)} \quad and \tag{30}$$

$$Q = \frac{\sigma}{(b - a)} \tag{31}$$

The beta distribution can assume many shapes, depending on the values of α and β. Shapes include uniform, triangular, skewed left or right, and even a shape resembling a normal distribution with finite limits, as shown in Figure 2-5. The beta distribution is similar to the normal distribution when $\alpha = \beta \geq 3$. Due to the complexities of beta distributions, they are not easily tabulated into a standard form such as the "Z" tables for the normal distribution. However, computer software is available to help with the construction of beta distributions and for calculating probabilities associated with these distributions. Harr (1987) provides the source code for this type of software.

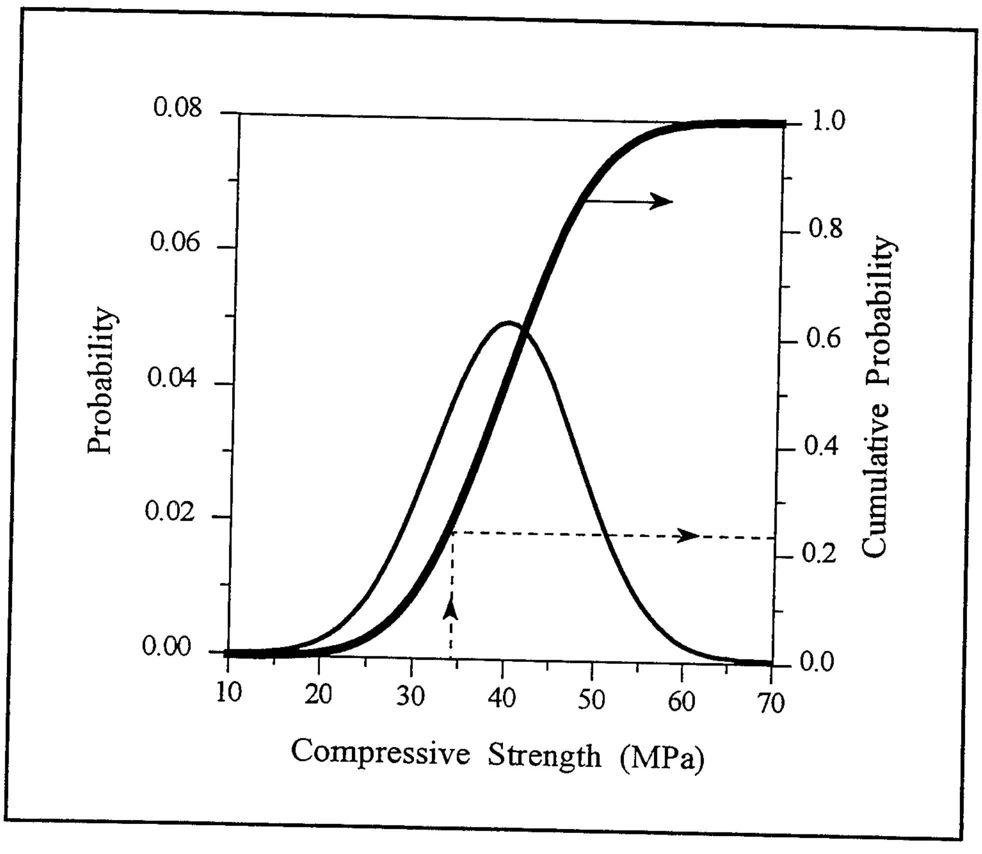

Figure 2-5. Concrete compressive strength with an assumed normal distribution

Example. Assume that a pavement engineer is interested in load transfer between slabs that rely solely on aggregate interlock. The engineer learns from literature that for his/her slab configuration the average load transfer is approximately 20 percent. He/she also learns that the standard deviation for load transfer is typically about 10 percent. Due to the fact that measurements cannot possibly fall below zero percent or above 50 percent and due to the fact that the distribution has been reported to be skewed, the engineer may decide that a beta distribution would provide a better representation of the load transfer measurements than a normal distribution. The beta distribution and the cumulative probability distribution for this problem were calculated with Harr's (1987) software, as shown in Figure 2-6. If the engineer was interested in the likelihood that load transfer would be less than 15 percent, a quick measurement taken from the cumulative probability plot would indicate that this should occur approximately 35 percent of the time.

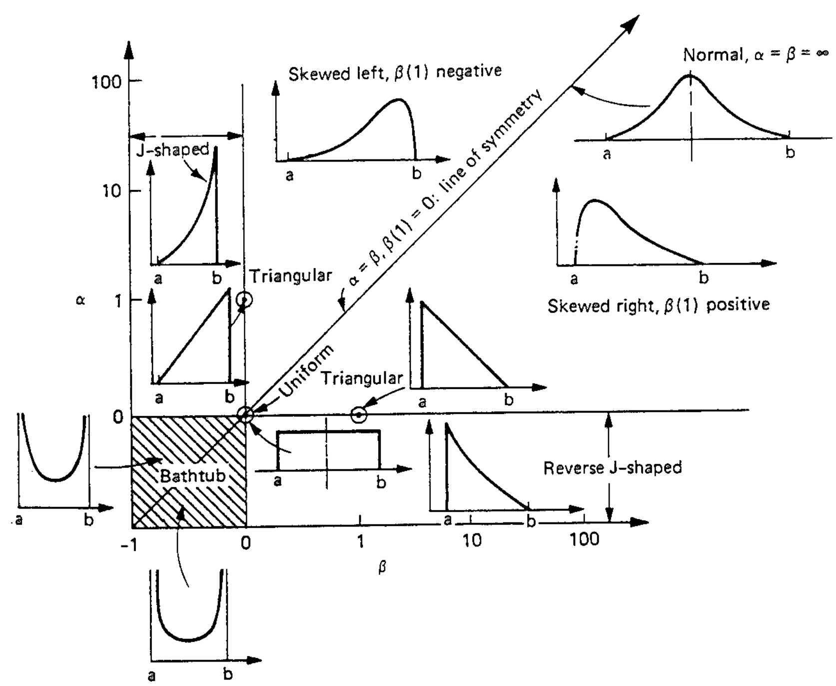

Figure 2-6. Shapes of the beta distribution (Harr 1987)

Sampling Distributions

The example problems used up to this point have included data sets that were considered to be "populations." When dealing with populations, the distribution parameters can be considered to be fixed values. When taking

samples from a population, however, the statistics that describe the distribution of sample values cannot be considered to be fixed. Different samples taken from the same population can generate different statistics. A statistic computed from a random sample is therefore a random variable. This inherent variability for sample statistics must be considered whenever samples are used to make inferences about a population.

All the possible values of a sample statistic can be described by a probability distribution for the statistic, often called a sampling distribution. Characteristics of a sampling distribution can be related to characteristics of the population from which the samples were drawn.

Assume samples that include n observations each are drawn from a population, $Y\,[=\{y\}]$, with mean μ and variance σ^2. As the number of samples drawn approaches infinity, the distribution of sample means, $\bar{y}_i$, will have a mean that approaches μ and a variance that approaches σ^2/n, as shown below.

$$\mu(\overline{y}_i) = \mu \qquad \textit{and} \tag{32}$$

$$\sigma^2(\overline{y}_i) = \frac{\sigma^2}{n} \tag{33}$$

This statement makes intuitive sense as one would expect the sample means to cluster around the population mean and one would expect the variance of sample means to be less than the variance of individual observations. Sample means are referred to as "unbiased" estimates of the population mean.

Distribution for sample means

An additional important characteristic of the distribution of sample means is the "central limit theorem." This theorem states that the distribution of sample means can be closely approximated by the normal distribution, regardless of the population from which the samples are drawn. The size of the samples required to validate this theorem is dependent on the shape of the parent population. If the population resembles normality, sample sizes of 10 or more should be sufficient. Sample sizes of 30 or more should be sufficient for populations of any other shape (Freund and Wilson 1993).

Grant and Leavenworth (1972) stated that even if sample size (n) is small, the distribution of the means of the samples can be very close to normal if the number of samples is sufficiently large. This theory holds true even if the parent population is far from normal. Grant and Leavenworth (1972) reported on a study in which 1,000 samples of size n = 4 were taken from two bowls: one containing numbered tags from a rectangular distribution population and one containing numbered tags from a triangular distribution

population. Neither of the original populations resembled a normal distribution, however, the distribution of sample means in each case was normal. They reported:

"The great practical importance of the normal curve arises even more from its use in sampling theory than from the fact that some observed distributions are described by it well enough for practical purposes. Of great practical significance is the fact that distributions of averages of samples tend to be approximately normal even though the samples are drawn from non-normal populations."

When the distribution of sample means can be assumed to be normal in shape, the standard normal variate Z can be used as a problem-solving tool. Recall the transformation of a random variable from a population to a standard normal variable.

$$z = \frac{y - \mu}{\sigma} \tag{34}$$

Similarly, a sample mean can be transformed from a population to a standard normal variable:

$$z(\overline{y}_i) = \frac{\overline{y}_i - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{35}$$

The denominator in Equation 30 represents the standard deviation for the distribution of sample means, which can be calculated from the variance of sample means (Equation 28), as shown below. The standard deviation of sample means is often referred to as the "standard error of the mean."

$$\sigma(\overline{y}_i) = \sqrt{\sigma^2(\overline{y}_i)} = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}} \tag{36}$$

Example. Suppose that an engineer is planning to construct a parking lot in an area that has a variable water table due to the presence of various types of soil. Suppose also that he/she knows from historical data that the depth to water around the site should follow a near-normal distribution. The mean and standard deviation of the depth to water is 2 meters and 0.5 meters, respectively. If the engineer is interested in the probability that a sample of 16 measurements will have a mean less than 1.75 meters, he/she can use the Z-table as shown below.

$$z(\overline{y}_i) = \frac{1.75 - 2}{\frac{0.5}{\sqrt{16}}} = -2 \tag{37}$$

From the standard Z table (Appendix A), the engineer could see that the probability of getting a z-value less than -2 is approximately 2 percent (0.0228). The engineer could then feel confident that the mean of a 16-replicate sample would very rarely be less than 1.75 meters.

For this same problem, assume the sample consists of only 4 replicates. Then the corresponding z-value is as shown below.

$$z(\overline{y}_i) = \frac{1.75 - 2}{\frac{0.5}{\sqrt{4}}} = -1 \tag{38}$$

From the standard z-tables, the engineer could see that the probability of getting a z-value less than -1 is approximately 16 percent (0.1587). With the decrease in sample size from n=16 to n=4, the standard deviation for the distribution of sample means doubled. As a result, the estimated probability for obtaining a given deviation from the mean increased.

The effects of sample size on the probability distribution for sample means is illustrated in Figure 2-7. In this figure, the distributions for sample means are shown to be more narrow (or tighter) than the standard normal distribution from which they were derived. As sample size increases, the distributions for sample means became progressively more narrow.

Distribution for sample variances

When samples of n observations each are drawn from a population with mean μ and variance σ^2, the sample variances s^2 can be used to construct a frequency distribution. The theoretical relationship between the mean of this distribution, $\mu(s^2)$, and the true population variance, σ^2, is shown below.

$$\mu(s_i^2) = \frac{n-1}{n}\sigma^2 \tag{39}$$

For small sample sizes, the mean of the distribution of sample variances would be less than the true population variance. However, as the sample size n increases, the mean of the distribution of sample variances becomes a good estimate for the true population variance. This inequality between $\mu(s^2)$ and σ^2 causes sample variances to be termed "biased" estimates of the population variance. In order to compensate for this bias, sample variances are calculated with $(n\text{-}1)$ as the divisor, rather than n (as shown below). As the

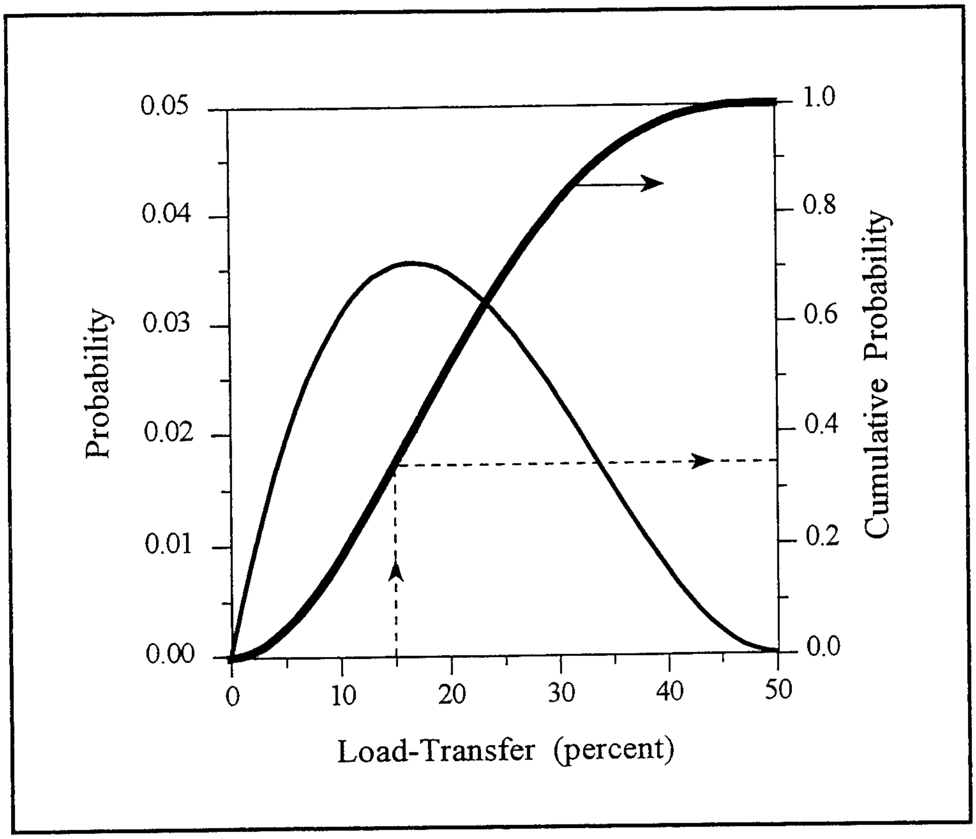

Figure 2-7. Load-transfer between pavement slabs with an assumed beta distribution

sample size (n) increases, the effect of the bias, along with the magnitude of the bias correction, decreases.

$$\textit{unbiased } s^2 = \frac{n}{n-1}(\textit{biased } s^2) \tag{40}$$

Recall the difference between the calculations for population variance σ^2 and sample variance s^2, as presented previously. While the divisor for population variance was N, the divisor for sample variance was n-1, as shown in Equations 41 and 42, respectively.

$$\sigma^2 = \frac{\sum_{i=1}^{N}(y_i-\mu)^2}{N} \tag{41}$$

$$s^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1} \tag{42}$$

The term *n-1* in these calculations for sample variance can be referred to as the degrees of freedom for the sample variance statistic. In general, the degrees of freedom for a statistic (ν) is defined as the number of independent observations (n) minus the number of population parameters (m) that must be estimated from sample observations in order to calculate the statistic. In symbols, $\nu = n\text{-}m$ (Spiegel 1990).

As shown previously, the calculation for sample variance requires an estimate for the population mean (μ), which is provided by the sample mean ($\bar{y}$). Therefore, the degrees of freedom for sample variance can be calculated as $\nu = n\text{-}m = n\text{-}1$.

T distribution

In previous applications of the *Z* tables, the standard deviation of the population (σ) was assumed to be known. In most practical situations, however, σ is not known and the only available measure of the population standard deviation is the sample standard deviation (s). In order to make continued use of the standard normal tables for sample mean distributions, one would like to substitute s for σ in the calculation of Z. This substitution can be made, but the resulting distribution is not exactly normal and has to be handled differently than the Z distribution. If s substitutes for σ, the distribution is referred to as a t distribution (or Student's t). The calculation of t is performed as shown in Equation 43. The t distribution is similar to normal in that it is bell-shaped and symmetric about its mean. Relative to normal, however, the t distribution is broader (has fatter tails). This breadth reflects increased dispersion for the variate, which is caused by the uncertainty of using a sample statistic to estimate the population standard deviation. As the sample size increases, confidence in the estimate of the population standard deviation increases, causing the t distribution to become more narrow and to approach normality (Baecher 1987), as shown in Figure 2-8.

$$t = \frac{\bar{y} - \mu}{\frac{s}{\sqrt{n}}} \tag{43}$$

The probability density function that describes the t distribution is tedious to integrate manually, so problems are typically solved with the help of computer software or standard t tables, such as that shown in Table A2 (Appendix A).

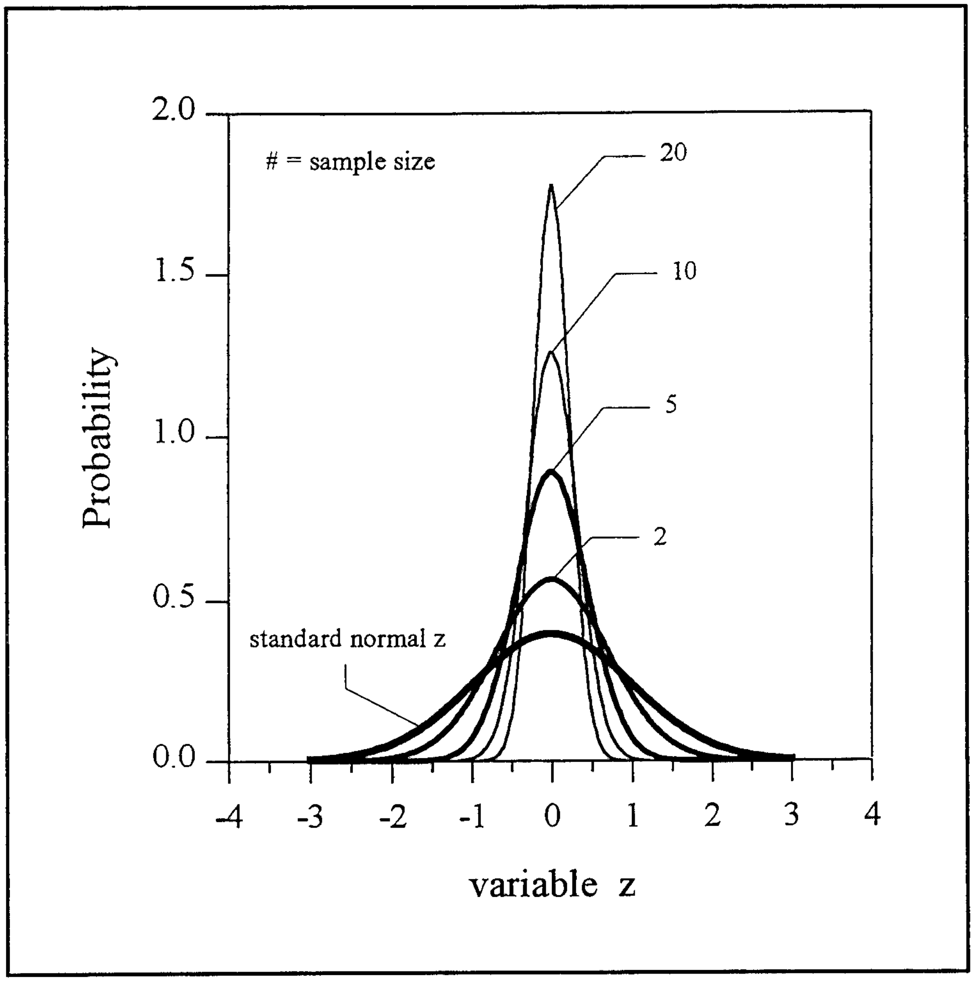

Figure 2-8. Effect of sample size on the probability distribution for sample means

Chi-squared (χ^2) distribution

The χ^2 distribution provides a method for comparing sample variances with population variances. If χ^2 is calculated for one or more samples taken randomly from a parent normal population, then χ^2 also becomes a random variable. Chi-square can be defined for each sample as shown below.

$$\chi^2 = \frac{SS}{\sigma^2} = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{\sigma^2} \tag{44}$$

where

y_i = each value within a sample

$\bar{y}$ = the calculated mean for the sample

SS = sum of squared deviations for the sample data

σ^2 = the variance for the parent population

n = the number of values obtained for the sample

The relationship between sample variance and population variance Equation 3), contained within the χ^2 variable, can be seen more clearly with a different expansion on the sum-of-squares term. Recall the definition for the unbiased estimate of population variance, calculated from a sample of data: $s^2 = SS/(n\text{-}1)$. Substitution of this relationship for the sum-of-squares in Equation 44 provides the following formula for χ^2.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \tag{45}$$

The probability density function that describes a χ^2 distribution is tedious to integrate manually, so software or published tables, such as that shown in Table A3 (Appendix A), are often used. Similar to the variances from which they are calculated, χ^2 values cannot be negative. The χ^2 distribution can be other than normal and its shape is dependent on the number of values obtained for the sample (n), as shown in Figure 2-9.

A few important characteristics of the chi-square distribution are listed below (Freund and Wilson 1993).

a. The mean of the χ^2 distribution is equal to the degrees of freedom, ν $(=n\text{-}1)$, and the variance is equal to 2ν.

b. For large values of ν (usually greater than 30), the χ^2 distribution can be approximated by the normal distribution with a mean and variance as described in "a." Thus, in these cases, the standard normal distribution can be used as shown below.

$$z = \frac{\chi^2 - \nu}{\sqrt{2\nu}} \tag{46}$$

c. The accuracy with which χ^2 represents SS/σ^2 for samples is not significantly affected when the parent population deviates slightly from normal. Severe departures from normality, however, can affect the reasonableness of using χ^2.

Example. Suppose that an agency is monitoring asphalt cement content for a paving job. If a sample of 10 measurements had a variance of 0.04 percent^2 and the variance had historically been 0.03 percent^2, what would be the probability of obtaining a sample variance of 0.04 percent or

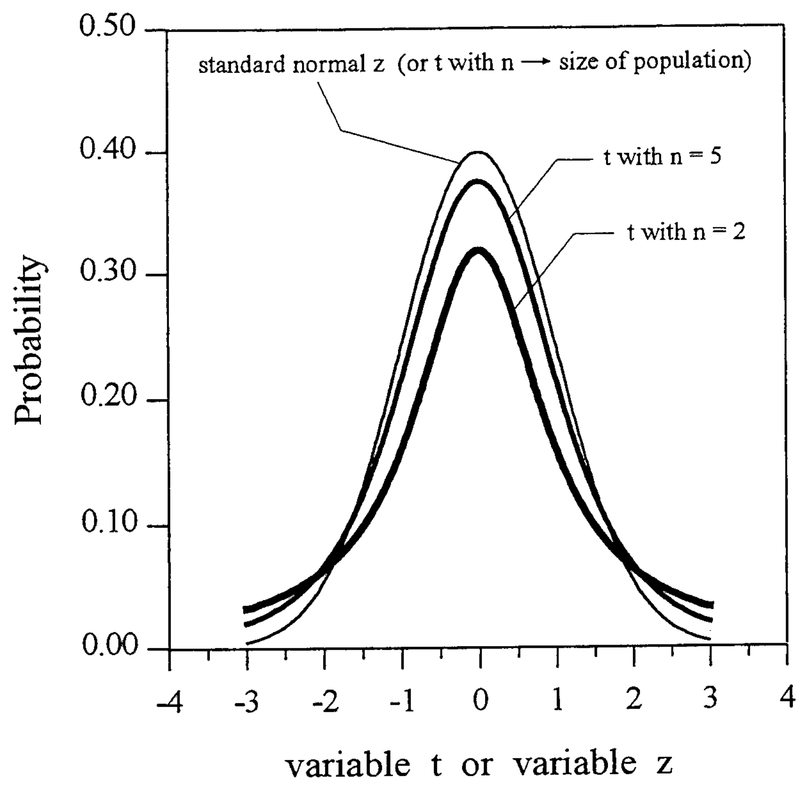

Figure 2-9. Effect of sample size on the probability distribution for student's *t*

higher? The χ^2 calculation is illustrated in Equation 42. Degrees of freedom is nine (*10-1*).

$$\chi^2 = \frac{(10-1)0.04}{0.03} = 12 \tag{47}$$

From the χ^2 distribution (Table A3) and linear interpolation, the probability of obtaining a χ^2 value of 12 or higher is found to be approximately 22 percent (0.2227). The evidence should suggest to the agency that this sample variance is not highly unlikely. Based on this data alone, there is no reason to judge that the manufacturing process for asphalt concrete is out of control.

F distribution

The *F*-statistic is defined as the ratio of two independent χ^2 random variables, each divided by their respective degrees of freedom. Assume that s_1^2 and s_2^2 are the variances of two independent samples of size n_1 and n_2, respectively. If these samples are taken from the same population or from two different populations with variances that are expected to be equivalent, then *F* can be calculated as shown below.

$$F = \frac{s_1^2}{s_2^2} \tag{48}$$

The choice for which variance estimate to place in the numerator is somewhat arbitrary. Typically, reference tables for the distribution of the F-statistic assume that the larger variance is used as the numerator. Since the samples from which variances are calculated are selected randomly, *F* is also a random variable. Similar to the χ^2 probability distribution, the *F* distribution is defined only for positive values. The shape of the *F* distribution is dependent on the degrees of freedom for each of the sample sets from which variance was calculated.

The probability density function for the F distribution is complex and would be difficult to integrate manually. Similar to other sampling distributions, the use of software or reference tables, such as that shown in Table A4 (Appendix A), are essential.

The *F* distribution is nonsymmetric, particularly when one or both of the distribution parameters has a low value. Figure 2-10 shows *F* distributions when the number of replicates for each sample are equal and consist of values ranging from 5 to 50. Figure 2-11 shows the effect on the *F* distribution when the number of replicates for one of the samples drops from 50 to 5.

The *F* distribution is used to conduct analysis-of-variance (ANOVA) procedures, which will be demonstrated in the next chapter. The *F* distribution can also be used to make simple inferences concerning the equality of variances for two normal populations.

Example. Suppose that two construction crews (A and B) are placing asphalt concrete surface course mixtures. Suppose further that 21 randomly located cores were removed from crew A's pavement while 11 randomly located cores were removed from crew B's pavement. The standard deviation for surface course thickness was determined to be 5 mm and 3 mm, respectively. If the two crews should be paving with equal uniformity in thickness, what is the probability of finding a difference in standard deviations as large or larger than that measured?

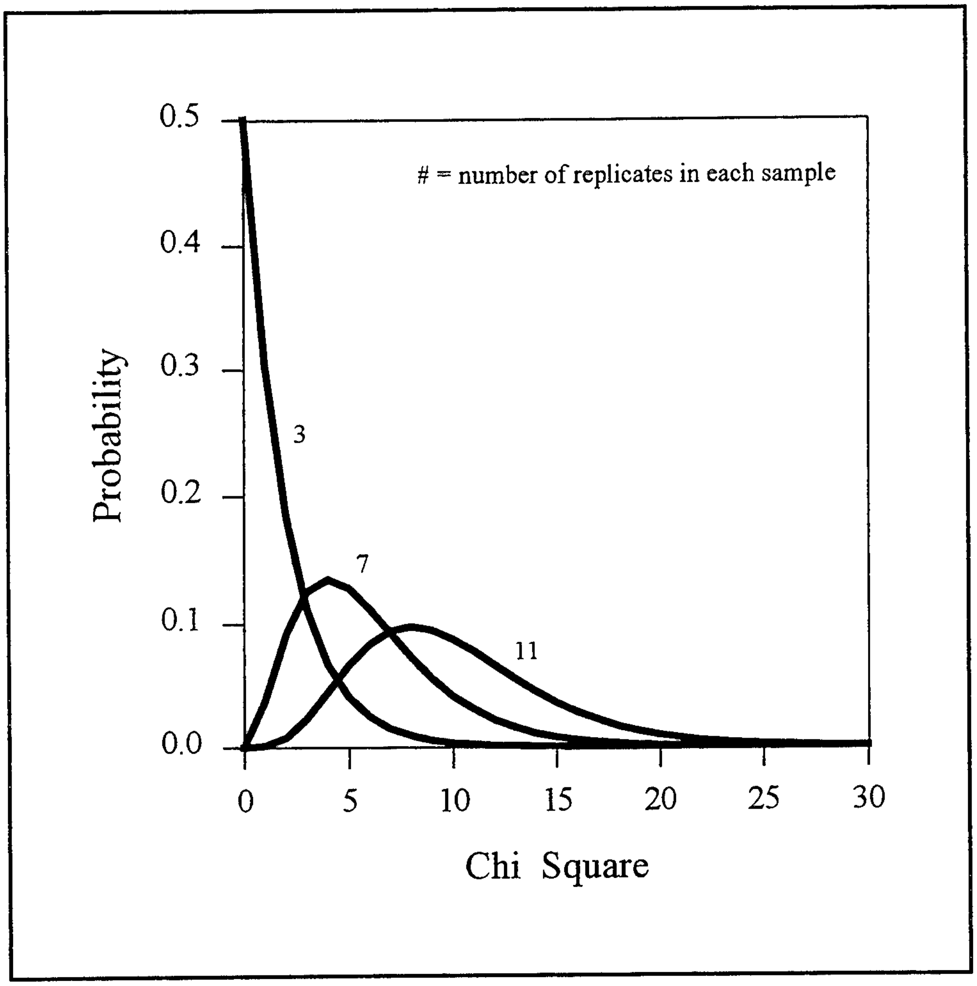

Figure 2-10. Effect of sample size on the chi-square distribution

The F-statistic is calculated as shown in Equation 49. The value 2.8 is compared to values provided in the standard F distribution table, with degrees of freedom equal to 20 and 10, $F\,(20,10)$. Using linear interpolation within Table A4, the probability of obtaining a ratio of variances this large is found to be less than 5 percent (0.0488). The variances can therefore be considered significantly different with a high degree of confidence. The project manager can feel confident that the pavement placed by crew B has a more uniform thickness than the pavement placed by crew A.

$$F = \frac{s_A^2}{s_B^2} = \frac{(5)^2}{(3)^2} = 2.8 \qquad (49)$$

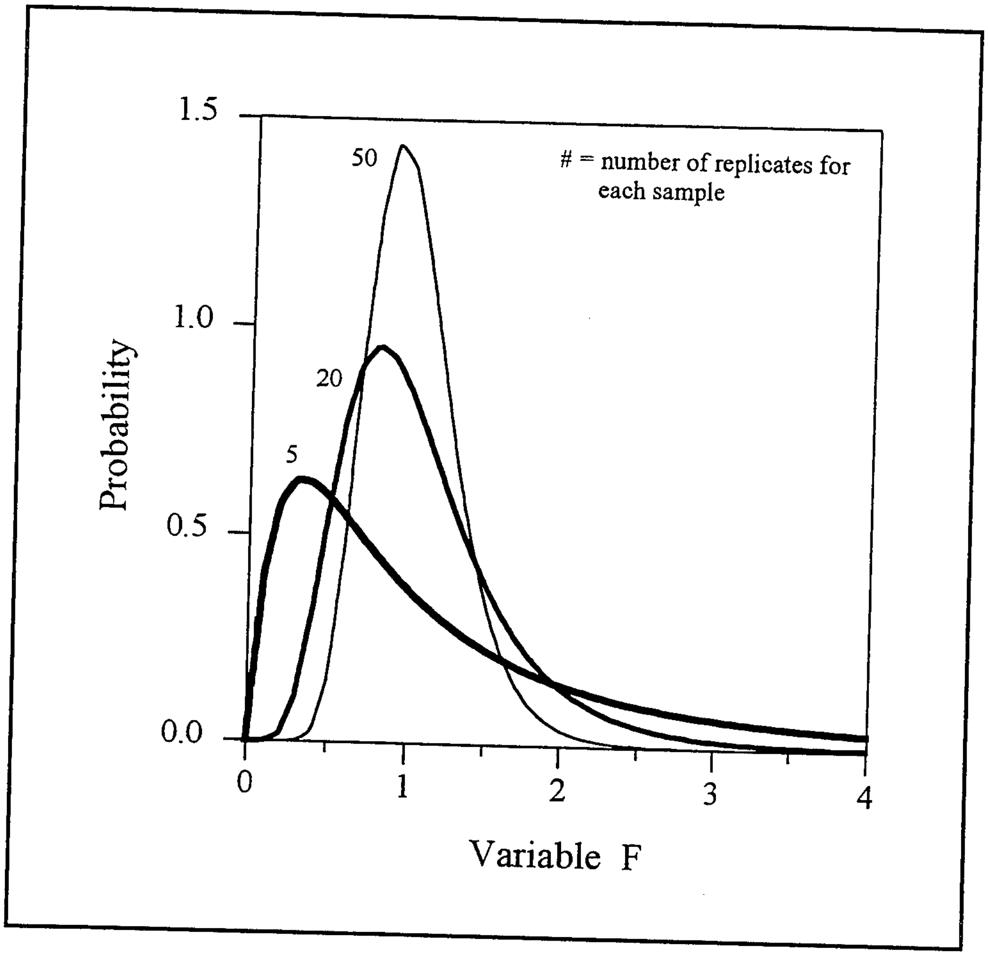

Figure 2-11. Effect of sample sizes on the F distribution

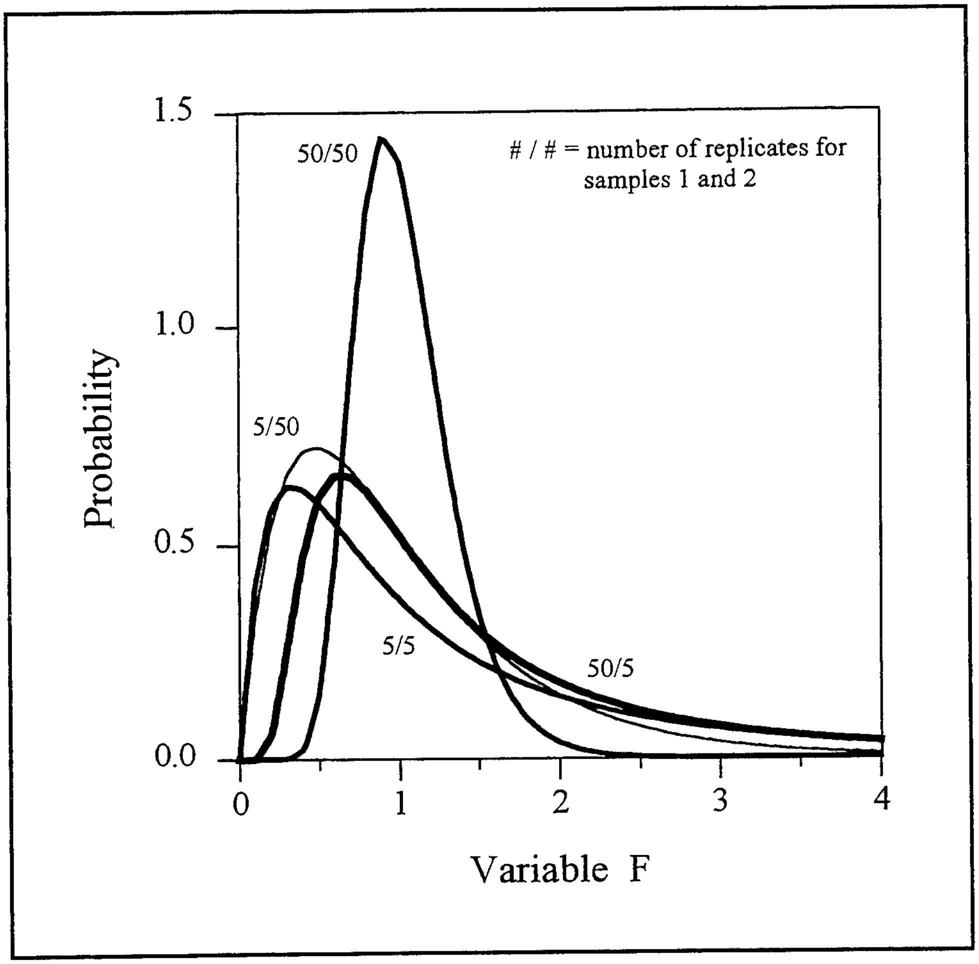

Figure 2-12. Effect of unequal sample sizes on the *F* distribution

3 Analyzing Pavement Materials Data

This chapter provides a review of techniques needed for studying the variability of construction data.

Distribution Shapes

Smooth curve approximations for the frequency distribution histograms for experimental data can take various forms (Willenbrock 1974), such as those shown in Figure 3-1. The symmetrical, or bell-shaped curve, is characteristic of the normal distribution. Knowledge of whether frequency distribution histograms are skewed is important because many statistical tests used for the interpretation of data are based on the assumption of symmetry, or even normality. The J-shaped and U-shaped distributions shown in Figure 3-1 are not common in cases for construction materials unless data has been manipulated. For example, compressive strength data for concrete can be reverse J-shaped if all strengths less than the specificied strength are discarded.

Symmetric and skewed distributions are examples of unimodal (single peak) distributions. Distributions can also be bimodal or multimodal if more than one peak exists. These characteristics have serious implications for statistical analyses because the sample of data has probably not been obtained from a single population. In a bimodal distribution, each of the two peaks may represent the mean of a different population.

The overriding pattern for the frequency distribution histograms for construction materials (e.g. quality control data) is that they tend toward bell-shaped curves, rather than J-shaped, U-shaped, or multimodal curves (Willenbrock 1974). When variability among data is caused by the cumulative effect of a large number of small perturbations or errors, the resulting frequencies of observations typically exhibit a near-normal distribution, which is bell-shaped (Baecher 1987, Benjamin and Cornell 1970). Even when the material property of interest does not have a normal frequency distribution, the frequency distribution of sample means is approximately normal (Baecher 1987). This idea is an element of the central limit theorem, as discussed in the previous chapter.

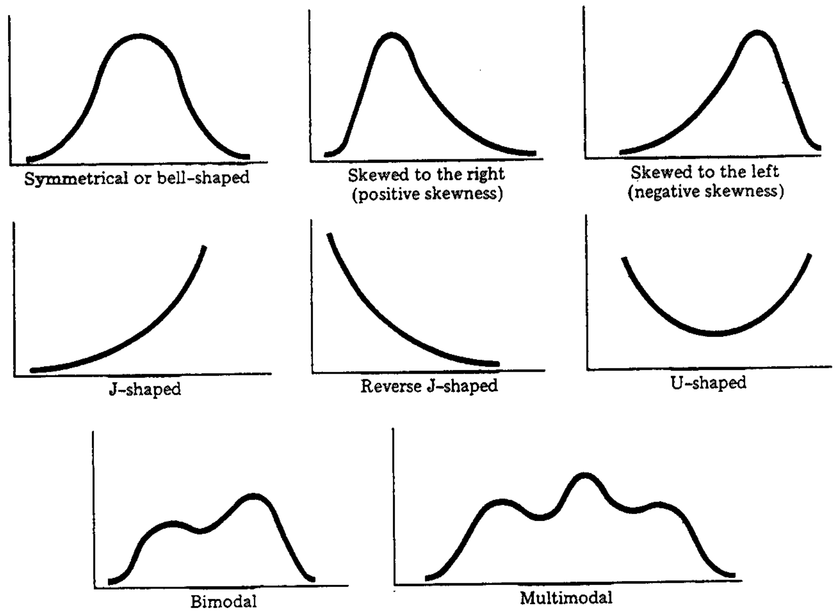

Figure 3-1. Characteristic shapes of smooth-curve approximations for frequency distributions (Spiegel 1988)

Departures From Normality

For the case where the frequency distribution for pavement materials data departs form the "normal" shape, the cause is probably one of the following (Rethati 1983):

a. Physical limits.

b. Errors caused by sampling and testing.

Physical limits means that the possible numerical values for a measured material property have either an upper limit, a lower limit, or both (Rethati 1983). For example, unconfined compressive strength for soil cannot be less than zero, so the results for low-strength soils would tend to have a positive skew. Physical limits that are not near the mean of a material property would tend not to induce skewness. The unconfined compressive strength of concrete cannot be less than zero, but average strengths are typically so far removed from zero that skewness is not induced by this numerical boundary. An example of potential negative skew occurs for measured degree of saturation for wet soil. If the mean degree of saturation is relatively large (greater than 60 percent), the distribution would tend to end abruptly at the physical limit of 100 percent.

To attain normality, the data forming a frequency distribution must typically come from a single population (Rethati 1983). If the material is concrete, all the concrete must be from the same mixture design. If the material is soil, horizontal and vertical survey boundaries must be employed so that the soil can be considered a single deposit. If two different types of soil are analyzed as a single population, a sampling error would have been committed, and the distribution for a material property may likely appear bimodal. If the property is plasticity index (PI) and the two soil types are clays of different plasticity, the measurements of one soil type may cluster around a PI of 30 (for example), while the measurements of the second soil type may cluster around a PI of 20. In order to evaluate this data properly, the two soil types should be treated as two different populations.

Additional sampling errors can be caused by deviations in the time of sampling. Time can permit changes in material properties, which can invalidate assumptions concerning the material's homogeneity, including the assumption that all the samples come from the same population (Rethati 1983). If the material is a silt soil and the measured property is moisture content, a day's difference in sampling time provides an opportunity for moisture content to change. If the material is an asphalt concrete and the measured property is temperature, measurements just after loading a truck should be considered a separate population from measurements upon arrival at the construction site.

Errors caused by testing can skew results (Rethati 1983). For soil, the level of disturbance imposed on a sensitive clay during sampling will affect shear strength measurements. If some of the samples are severely disturbed, they will exhibit lower strengths, which will cause the distribution to skew in the negative direction. If some concrete cylinders are allowed to dry before measuring compressive strength, their measured strength will be high, which will cause the distribution to skew in the positive direction.

Tests for Normality

Many common statistical procedures assume that the frequency distribution of data is Gaussian, or normal. It is important to know when incorrect assumptions for normality can lead to gross errors and to know how to check for normality. Incorrect assumptions concerning the shape of probability distributions are most critical when the statistical analysis is concerned with the "tails" of the distribution, or the very low and very high values (Coleman and Steele 1989). For example, the use of statical methods for the determination of outliers in a data set involves the tails specifically. These methods are sensitive to the shape of the probability distribution. Statistical methods that use the entire body of data, such as methods for comparing sample means, are less sensitive to the shape of the probability distributions.

The simplest method for determining whether or not a sample frequency distribution is normal involves visual inspection of the shape of its histogram.

If the histogram is symmetric and bell-shaped, the assumption of normality is probably acceptable.

The next level of sophistication for testing normality is another graphical alternative, involving the production of a "normal probability plot." Several types of normal probability plots exist, but they all share a common characteristic: a distribution that is normal or nearly normal will plot as a straight line, extending from the lower left quadrant to the upper right quadrant (Willenbrock 1974c). Examples of these plots will be shown as part of an example problem later in this chapter. A popular style used by statistical software packages is to format the x- and y-axes as unitless linear scales. The x-axis is used for representing all real data in terms of standard deviations from the mean. The y-axis is used for representing standard normal values (Z), obtained using the cumulative percent frequencies for the data. For example, if an observation is greater than 50 percent of the entire data set, its expected normal Z would be 0.0 (Table A1). If an observation is greater than 80 percent of the entire data set, its expected normal Z would be approximately 0.84. If the data is normal, its calculated standard deviations from the mean should correlate positively with the standard normal deviations from the mean (Norusis 1993). These plots are typically referred to as Z-plots. An example problem is used later in this chapter to demonstrate the construction of a Z-plot.

Another version of the normal probability plot uses linear scales for the x- and y-axes, with percent as their units. The x-axis is used to represent cumulative percent frequencies for the actual data. Each data point is converted to standard deviations from the mean and this value is used in the standard normal Z table to obtain an expected cumulative percent frequency, which is plotted on the y-axis. If the data is normal, its calculated cumulative percentages should correlate positively with the standard normal cumulative percentages (Norusis 1993). These plots are typically referred to as P-plots. An example problem is used later in this chapter to demonstrate the construction of a P-plot.

Several methods exist for purely quantitative testing of normality. Collectively, these methods are referred to as "goodness-of-fi" tests. In each method, the decision of normality is reduced to statistical testing of the null hypothesis, which states that the distribution is normal in shape. Rejection or acceptance of this hypothesis involves a comparison between a calculated statistic and a test statistic. These methods include the chi-square (χ^2) test, the Kolmogorov-Smirnov test, the Lilliefors test, and the Shapiro-Wilks test. These methods vary in terms of their leniency towards rejecting the null hypothesis of normality. It is important to realize that as data sets become large, each of these goodness-of-fit tests becomes hard to satisfy (Norusis 1993). Frequency distributions for real-world data will rarely exhibit perfect normality, even if they appear to be symmetric and bell-shaped.

In order to account for the inherent strictness of normality testing for large datasets, the probability for falsely rejecting normality should be decreased, essentially making the normality test more lenient. The probability of falsely rejecting normality is controlled by setting a level of test

significance (α). Practitioners who test for normality by these quantitative methods should supplement their results with normal probability plots. This practice will enable the practitioners to develop a sense of the relationship between dataset size, normality, and appropriate level of test significance (α).

Example. The χ^2 "goodness-of-fit" test is demonstrated in this example because it is commonly used and it is simple. The data are moisture content determinations for a residual, high-plasticity clay (Table 3-1). There are 229 measurements, with a mean of 31.9 percent and a standard deviation of 12.8 percent. To begin a chi-square test, the data are categorized into intervals, as if producing a frequency distribution plot. Distances of the intervals from the mean are determined from the upper end of the intervals and are expressed in terms of standard deviations. This is necessary so that the actual data can be compared to expected values obtained from the standard normal table (Table A1). The standard normal table provides the probability of obtaining a sample value within each class interval, if the data was normally distributed. Expected observation frequencies for each class interval, assuming normality, can then be obtained by multiplying each class interval probability by the total number of observations. The contribution to chi-square from each class interval is calculated as:

$$\chi^2 \; contribution = \frac{(observed - expected)^2}{expected} \tag{50}$$

The total χ^2 is then the sum of all the class interval contributions. For this example, the total chi-square (sum of column F) was 10.52.

The degrees of freedom for chi-square is the total number of class intervals (k) minus one, minus the number of independent population parameters estimated (m). The subtraction of one from k, in addition to subtracting m, accounts for a constraint condition. The constraint states that if k-1 expected frequencies are known, the remaining frequency can be determined (Spiegel 1990). For this example, there were 14 class intervals and we estimated two population parameters: mean (=31.9 percent) and standard deviation (=12.8 percent). Therefore, we had 11 (14-1-2) degrees of freedom. From Table A3, we can determine the probability of obtaining a chi-square larger than 10.62 if the population was normal. Using linear interpolation, we find this probability to be 0.48 or 48 percent. Therefore for this example we cannot reject the assumption of normality. The assumption of normality should be rejected only if the probability of obtaining a χ^2 larger than the calculated value is less than the established level of significance. For this example, if the level of significance had been set at 5 percent, the assumption of normality could only have been rejected if the probability of obtaining a χ^2 larger than the calculated value was less than 5 percent.

Using the same set of data, values for both a Z-plot and a P-plot were calculated, as shown in Table 3-2. Values for these plots can be obtained using either individual data points or class intervals. Statistical software packages will typically use all the data points. For simplicity, the class intervals were

Table 3-1
Goodness-of-Fit for Moisture Contents (Percent) of Residual, High-Plasticity Clay (adapted from Steel and Torrie 1980)

Class Interval	A	B	C	D	E	F
0.6 to 5.5	7	-26.43	-2.065	0.0194	4.44	1.48
5.6 to 10.5	5	-21.43	-1.674	0.0277	6.34	0.28
10.6 to 15.5	7	-16.43	-1.284	0.0525	12.0	2.08
15.6 to 20.5	18	-11.43	-0.893	0.0863	19.8	0.16
20.6 to 25.5	32	-6.43	-0.502	0.1219	27.9	0.60
25.6 to 30.5	41	-1.43	-0.112	0.1477	33.8	1.53
30.6 to 35.5	37	3.57	0.279	0.1545	35.4	0.07
35.6 to 40.5	25	8.57	0.670	0.1386	31.7	1.42
40.6 to 45.5	22	13.57	1.060	0.1068	24.5	0.26
45.6 to 50.5	19	18.57	1.451	0.0712	16.3	0.45
50.6 to 55.5	6	23.57	1.841	0.0405	9.27	1.15
55.6 to 60.5	6	28.57	2.232	0.0201	4.60	0.43
60.5 to 65.5	3	33.57	2.623	0.0084	1.92	0.61
65.6 to 70.5	1	38.57	3.010	0.0044	1.01	0.00

A = Observed frequency.
B = Deviation of high endpoint from mean.
C = Standard deviations from mean.
D = Normal probability of obtaining a value in the class interval (Table A1).
E = Expected frequency, based on normality.
F = Contribution to chi-square [(observed-expected)^2/expected].

used in this example problem. The plots are shown in Figures 3-2 and 3-3. Based on the results from the χ^2 analysis, one would expect the plots to show a strong positive correlation between the data and the standard normal values. The plots support this expectation and show that the data are at least nearly normal.

Transformations

If a data set is found not to be normally distributed, two options exist for transforming the data into a new data set that may be normally distributed. The first option is to obtain or calculate new data that have similar engineering value. For example, California bearing ratio measurements for in-situ clay may not be normally distributed, but plate-bearing load test results for the same clay may be normal in shape.

Table 3-2
Z-Plot and P-Plot Calculations for Moisture Contents of Residual, High-Plasticity Clay (adapted from Steel and Torrie 1980)

Class Interval	Observed Frequency	Z-Plot		P-Plot	
		x-axis[1]	y-axis[2]	x-axis[3]	y-axis[4]
0.6 to 5.5	7	-2.065	-1.866	3.1	1.9
5.6 to 10.5	5	-1.674	-1.626	5.2	4.7
10.6 to 15.5	7	-1.284	-1.385	8.3	10.0
15.6 to 20.5	18	-0.893	-0.986	16.2	18.6
20.6 to 25.5	32	-0.502	-0.519	30.1	30.8
25.6 to 30.5	41	-0.112	-0.050	48.0	45.6
30.6 to 35.5	37	0.279	0.364	64.2	61.0
35.6 to 40.5	25	0.670	0.678	75.1	74.9
40.6 to 45.5	22	1.060	1.024	84.7	85.5
45.6 to 50.5	19	1.451	1.476	93.0	92.7
50.6 to 55.5	6	1.841	1.706	95.6	96.7
55.6 to 60.5	6	2.232	2.120	98.3	98.7
60.5 to 65.5	3	2.623	2.650	99.6	99.6
65.6 to 70.5	1	3.010	N/A	100	99.9

[1] Class intervals represented as standard deviation from the mean (real data).
[2] Normal standard deviation from the mean (based on real cumulative percent frequency).
[3] Cumulative percent frequency (real data).
[4] Normal cumulative frequency as a percent (based on real standard deviation from the mean).
N/A Not applicable.

The second option for transforming data into a normal distribution involves recomputing all data with either a square root, a logarithm, or a trigonometric function. If a data set is lognormal in shape, taking the logarithm of all data points should create a normal distribution. Conversely, taking the logarithm of normally distributed data can cause the distribution to deviate from normality. While soil porosities (p) and void ratios (e) are often normally distributed, log(p) and log(e) are typically not (Schultze 1972). As an example of a trigonometric transformation, researchers found that the angle of internal friction (ϕ) for sand could be transformed into a normal distribution by applying the cotangent function (Schultze 1972).

Example. Suppose an airfield runway needs to be evaluated for its allowable passes by a fully-loaded F-15 aircraft. Suppose further that 119 estimates for allowable passes have been obtained from non-destructive evaluations using a falling-weight deflectometer. An example data set, organized by class intervals, is shown in Table 3-3. These data were evaluated

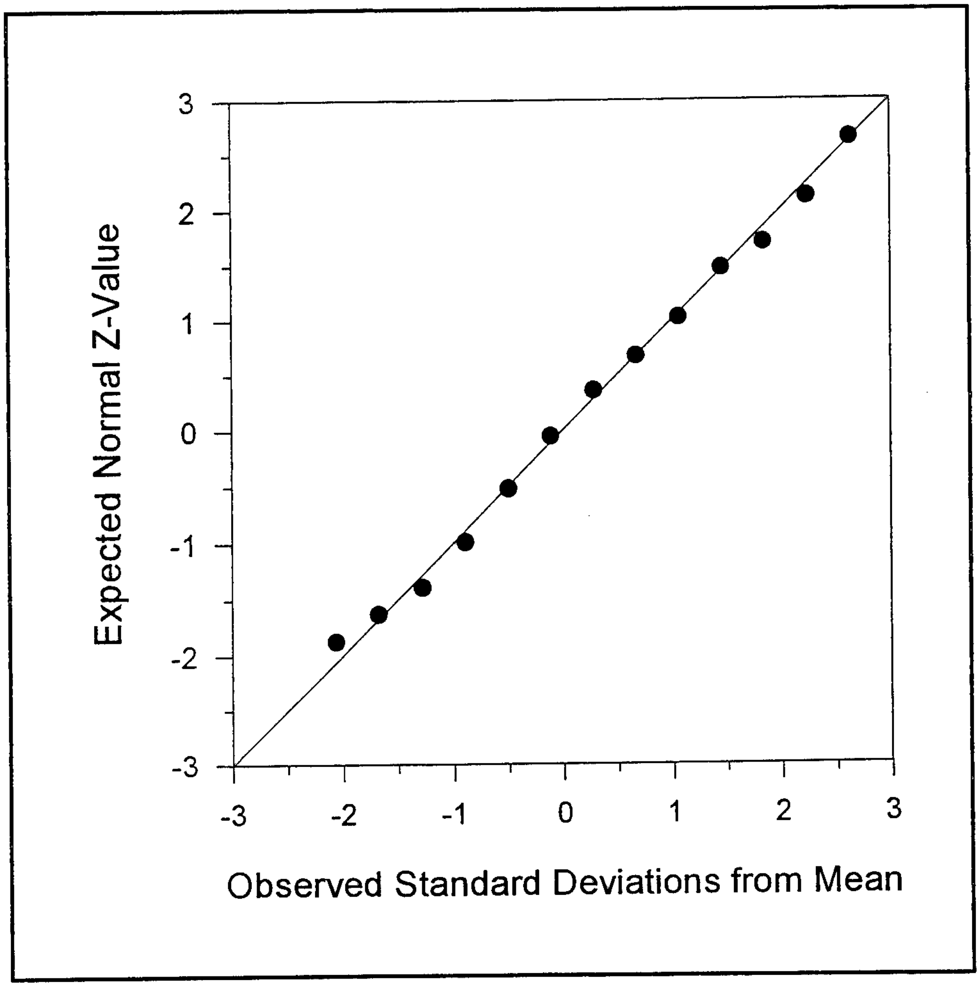

Figure 3-2. Z-Plot for a example problem concerning the moisture content of soil

for normality using both the χ^2 test and the Kolmogorov-Smirnov test. The χ^2 test did not reject normality, but the Kolmogorov-Smirnov test rejected normality at a significance level of 5 percent. These data, which are shown in Figure 3-6, appear to have a positive skew. This was confirmed by calculating their coefficient of skew (= 0.62).

Distributions for allowable passes are often lognormal in shape, so transformations using the logarithmic function can often improve normality. The transformed allowable passes data are also shown in Table 3-3. Neither the χ^2 test or the Kolmogorov-Smirnov test rejected normality for these transformed data. These data, which are shown in Figure 3-7, do not appear to have any significant skew. This was confirmed by calculating their coefficient of skew (= -0.07).

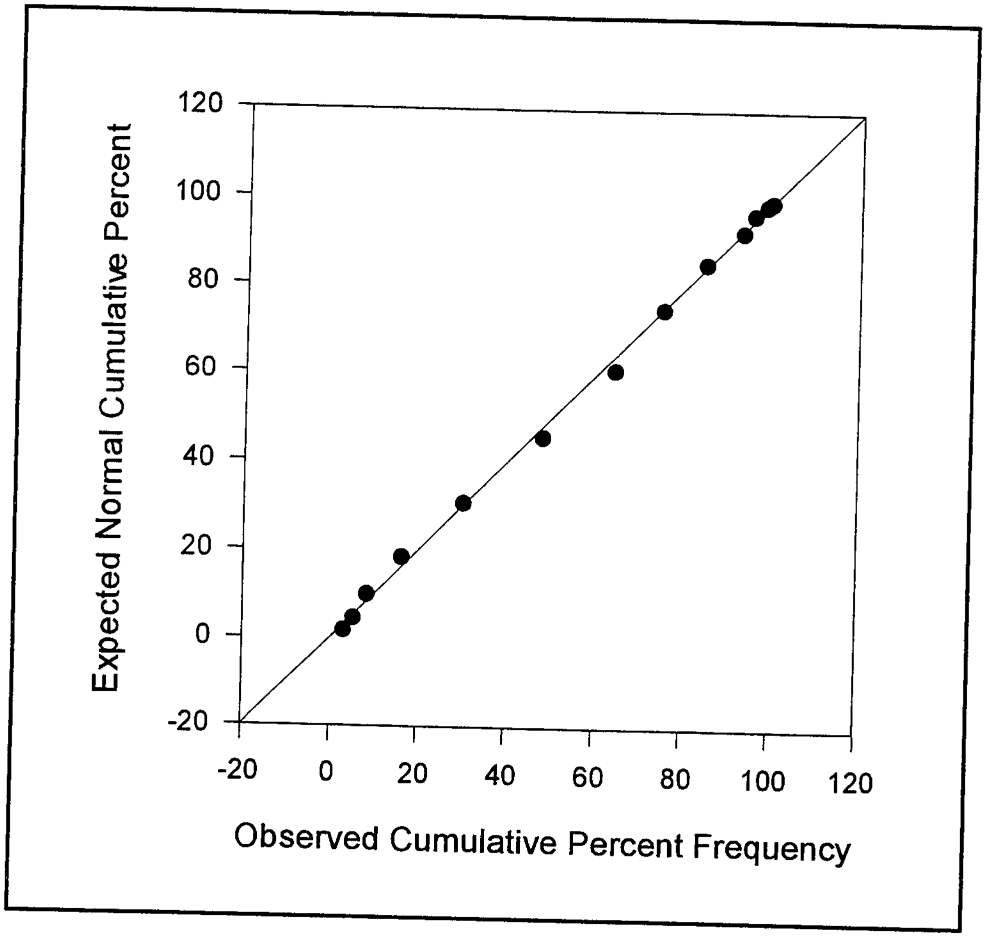

Figure 3-3. P-Plot for an example problem concerning the moisture content of soil

Handling Potential Outliers

In handling large quantities of data, some results may appear to be isolated and much different than the rest of the population. If the physical reason for the odd results is known, these data may be (Rethati 1983):

a. Discarded.

b. Discarded and replaced.

c. Corrected on a physical basis.

Corrections on a physical basis involve the use of physical laws or known physical relationships to correct sampling and testing errors.

Table 3-3
Transformation of Allowable Passes Data for an Airfield Feature

Allowable Passes		Log (Allowable Passes)	
Class Intervals	Observations	Class Intervals	Observations
181000-200000	3	5.26-5.28	1
201000-220000	2	5.29-5.31	2
221000-240000	5	5.32-5.34	2
241000-260000	6	5.35-5.37	4
261000-280000	9	5.38-5.40	5
281000-300000	14	5.41-5.43	7
301000-320000	16	5.44-5.46	12
321000-340000	21	5.47-5.49	16
341000-360000	10	5.50-5.52	21
361000-380000	11	5.53-5.55	16
381000-400000	5	5.56-5.58	11
401000-420000	5	5.59-6.61	8
421000-440000	4	5.62-6.64	6
441000-460000	2	5.65-5.67	3
461000-480000	2	5.68-5.70	3
481000-500000	1	5.71-5.73	1
501000-520000	1	5.74-5.76	1
521000-540000	1		
541000-560000	1		

Example. Suppose a concrete ready-mix plant performs quality control testing with 152x305-mm cylinders. Suppose further that a few data points in a month's production appear to be outliers (or extreme values), well above the mean. The concrete supplier may discover that these data were obtained with 76x152-mm cylinders. The supplier may choose to factor down the strength measurements with known relationships between the strengths measured by the two cylinder sizes. Whether discarding or correcting, all activities should be documented.

If the physical reason for outliers is not known, the outliers may only be removed from the data set with proper statistical procedures. Even with these tools, data removal should be performed sparingly and should be documented. Removing data can be counterproductive if the true dispersion or shape of a distribution is lost. If the apparent outliers are actually

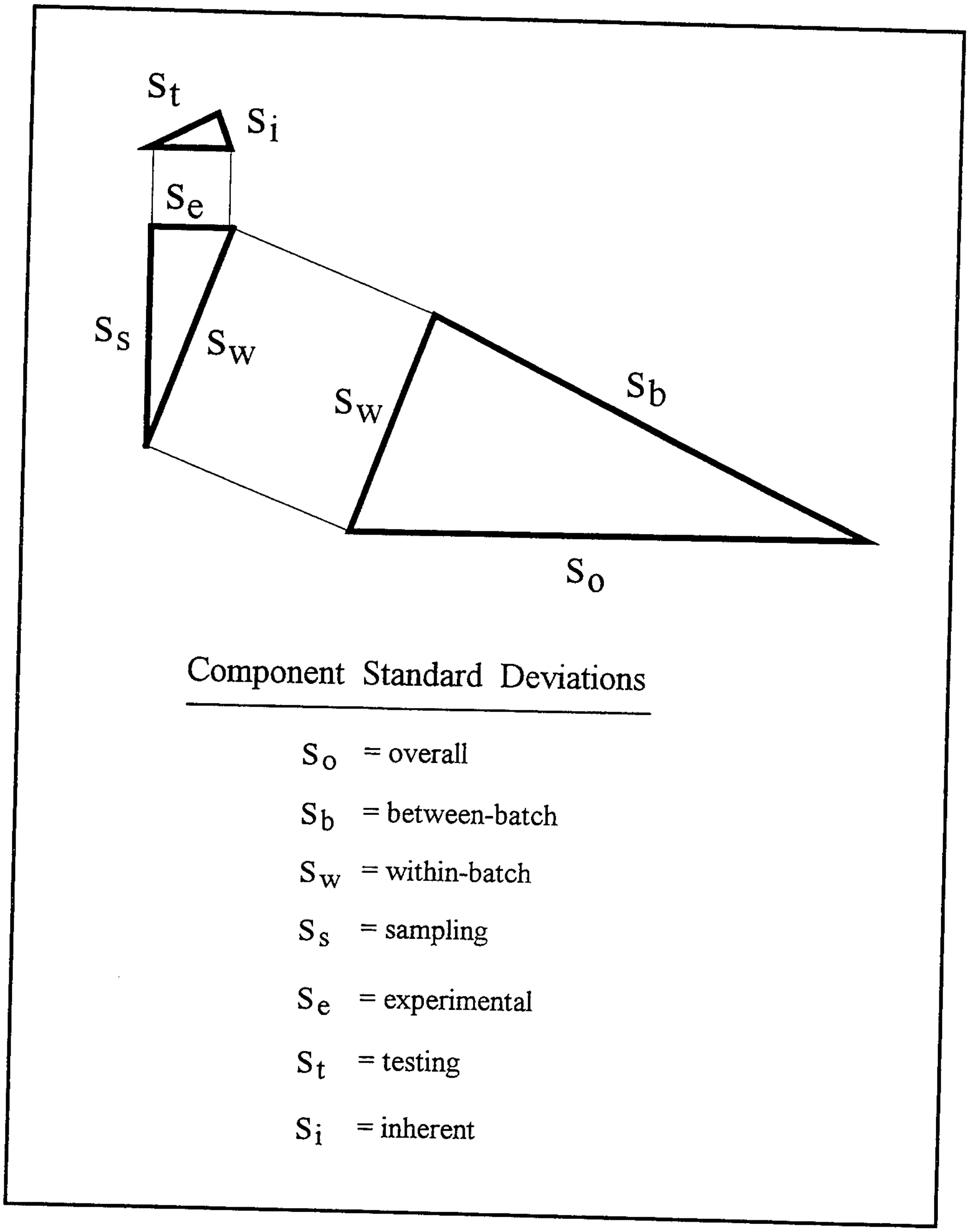

Figure 3-4. Variance components for a construction site study

reflections of a data set's skewness, then the removal of the outliers would provide a false image of symmetry.

When considering removal of outliers by statistical methods, all possible physical reasons for the outlying data should be investigated. The sampling procedure should be reviewed to ensure that the outlying data came from the same population as the other data. Test procedures should be reviewed to ensure that the outlying data was tested in the same manner as the rest of the observations. Finally, data analyses should be reviewed to ensure that there were no calculation errors. Once these efforts have been exhausted, the next

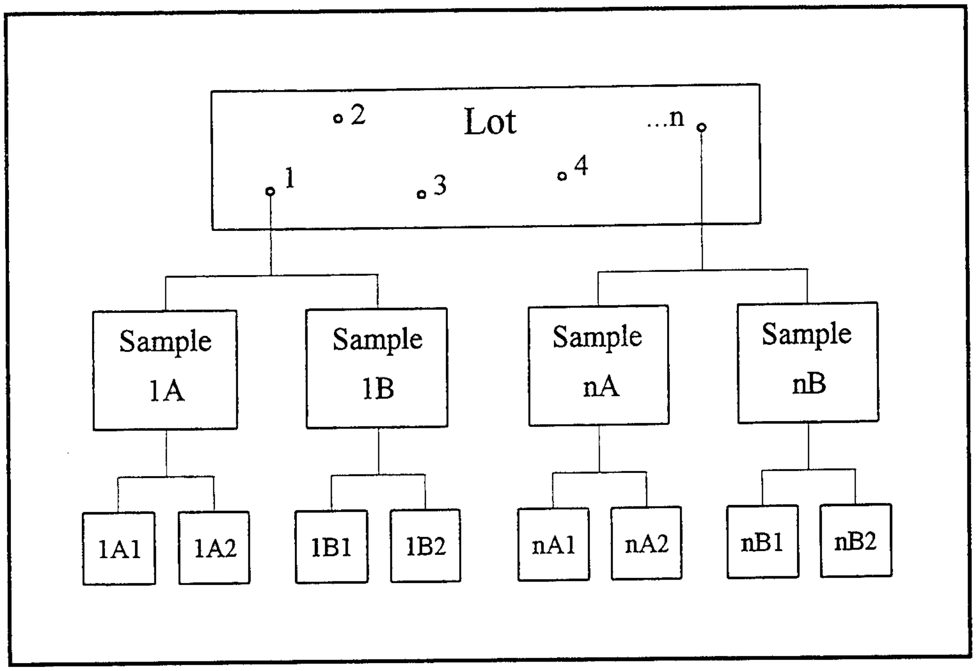

Figure 3-5. Sampling plan for studying construction site variability

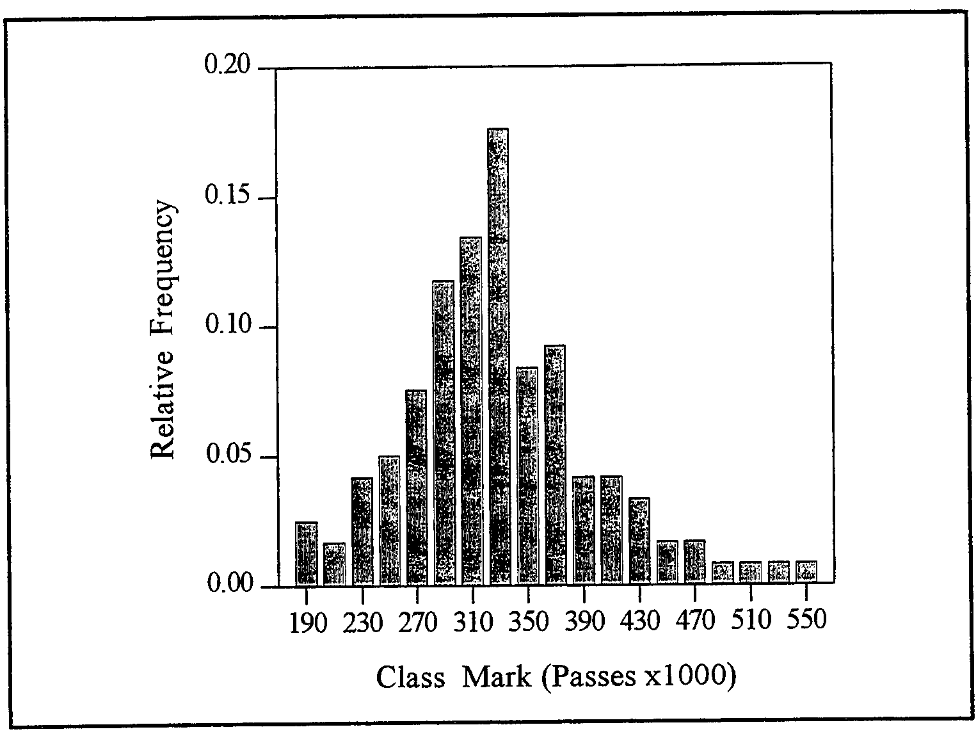

Figure 3-6. Frequency distribution for allowable aircraft passes

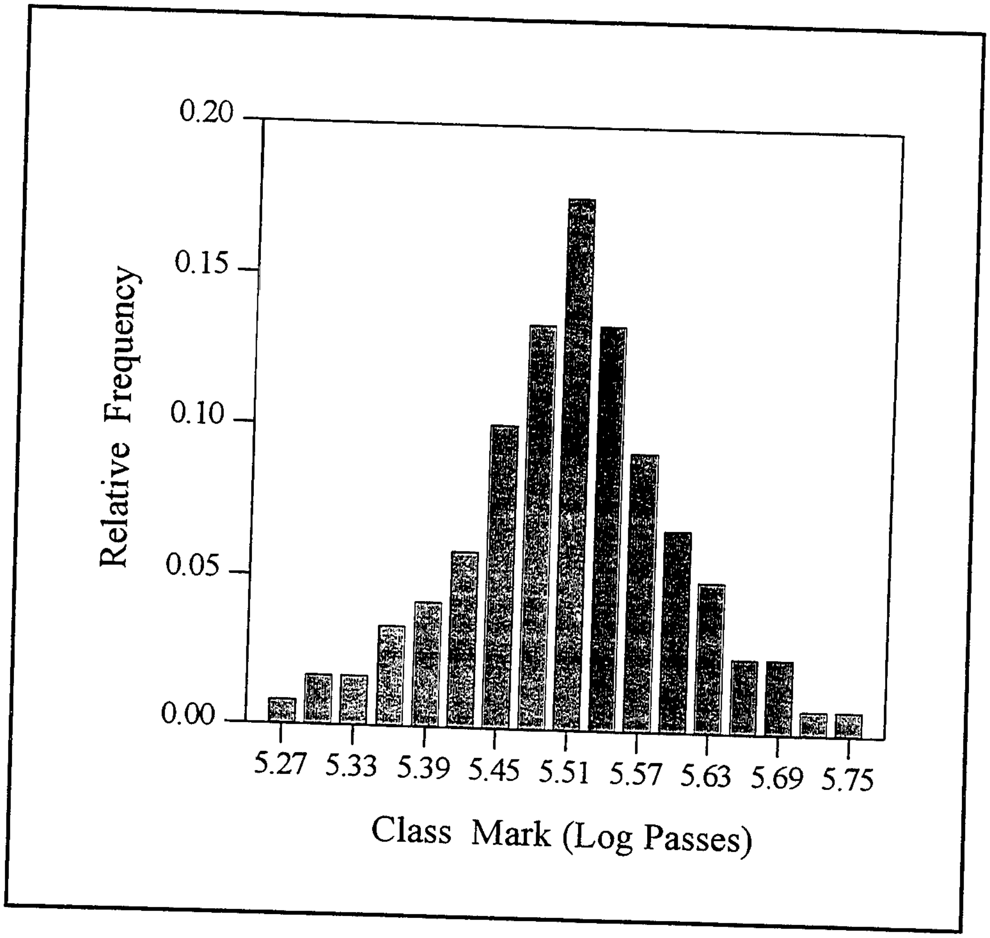

Figure 3-7. Frequency distribution for transformed (log) allowable passes

step is to determine whether the distribution is normal or other shape. The shape of a distribution can influence the choices for oulier testing. Published methods for determining outliers include the T-statistic, the Tietjen-Moore statistic, and Chauvenot's criterion. The T-statistic, which is calculated differently than Student's t-statistic discussed in Chapter 2, can only be applied to normal populations and can only handle a single outlier. The Tietjen-Moore statistic can also only be applied to normal populations, but it can test for multiple low and high outliers. Chauvenot's criterion can detect multiple outliers and it can be applied to distributions that are other than normal.

The T-statistic, for use with a single outlier, requires conversion of the potential outlier to a test criterion ($T_{calculated}$):

$$T_{calculated} = \frac{|y_i - \bar{y}|}{s} \tag{51}$$

where

y_i = the potential outlier

$\bar{y}$ = the sample mean

s = standard deviation for the sample.

This test criterion, $T_{calculated}$, is compared to standard T-values (Table A5) to determine the probability of obtaining a deviation of this magnitude, relative to the sample mean. When using the standard T-values, the engineer would select a level of significance, which will define the probability of falsely discarding a data point as an outlier. According to guidelines in ASTM (1995b) E 178, levels of significance of 5 percent or less are generally advisable for testing outliers. If $T_{calculated}$ is larger than the standard T-value at the selected level of significance, the potential outlier may be discarded (Schiff and D'Agostino 1996).

Example. Assume that the following data represents extracted asphalt cement contents from a single lot: 5.68, 5.70, 5.70, 5.70, 5.72, 5.72, 5.72, 5.78, 5.84, and 5.96. The mean and standard deviation for this data is 5.75 percent and 0.09 percent, respectively. If we question the validity of the measured value 5.96, we would calculate T as (5.96-5.75)/0.09 = 2.33. Inspection of Table A5 reveals that the probability of occurrence for a T-value this high is approximately 2 percent. If we had chosen a significance level of 5 percent, we would conclude that the extreme value did not come from the same population as the rest of the data and it would be discarded.

The Tietjen-Moore (T-M) statistic can be applied in cases where multiple outliers exist on either the low side of the data, the high side of the data, or on both sides of the data (Tietjen and Moore 1972). Let the sample values be $y_1, y_2, y_3, \ldots, y_n$ and compute the sample mean, $\bar{y}$. Then compute the n absolute residuals:

$$r_1 = |y_1 - \bar{y}|,\ r_2 = |y_2 - \bar{y}|,\ \ldots,\ r_n = |y_n - \bar{y}| \tag{52}$$

Now relabel the original observations, $y_1, y_2, y_3, \ldots, y_n$, as z's in such a manner that z_i is that observation whose absolute residual is the i^{th} largest. Relabeled observation z_1 should have the smallest residual and relabeled observation z_n should have the largest residual. The Tietjen-Moore statistic for testing the significance of the k largest residuals is then:

$$E_k = \frac{\sum_{i=1}^{n-k} (z_i - \bar{z}_k)^2}{\sum_{i=1}^{n} (z_i - \bar{z})^2} \tag{53}$$

where

$$\bar{z}_k = \sum_{i=1}^{n-k} \frac{z_i}{(n-k)} \tag{54}$$

The term $\bar{z}_k$ represents the mean of the $(n\text{-}k)$ least extreme observations and the term $\bar{z}$ is the mean of the full sample. If the calculated value of E_k is smaller than the critical value of E_k, shown in Table A6 (Appendix A), then the outlier can be discarded. As with the T-statistic, the engineer will have to select a level of significance (α), which will define the probability of falsely discarding a potential outlier.

For the example data shown in Table 3-4, a T-M statistic can be calculated for each of three different cases: one outlier (E_1), two outliers (E_2), and three outliers (E_3). The calculated T-M statistic for each case is shown in Table 3-5.

Table 3-4
Percent of Particles Finer Than the No. 200 Sieve for Asphalt Concrete Aggregate

Observation (Percent)	Absolute Residual (r_i)	$(z_i-\bar{z})^2$
6.1	0.21	0.0441
6.0	0.11	0.0121
5.8	0.09	0.0081
5.7	0.19	0.0361
5.9	0.01	0.0001
5.5	0.39	0.1521
5.4	0.49	0.2401
5.6	0.29	0.0841
6.0	0.11	0.0121
6.9	1.01	1.0201

Average observation ($\bar{z}$) = 5.890 percent.
$\sum (z_i-z)^2$ = 1.609 percent^2.

The critical T-M statistics for this problem can be obtained from Table A6. Assuming that the chosen level of significance is 5 percent, the T-M statistic for 10 observations for one, two, and three outliers is 0.353, 0.172, and 0.083, respectively. An observation can be considered an outlier if the calculated T-M statistic is less than the critical T-M statistic. Therefore, the data set has only one outlier. This outlier would be the observation with the largest absolute residual, which is 6.9 percent.

Table 3-5
Execution of the Tietjen-Moore Statistic for Determining of Outliers

Number of Outliers	T_k	E_k
1	5.778	0.476/1.609 = 0.296
2	5.825	0.315/1.609 = 0.196
3	5.871	0.194/1.609 = 0.121

Chauvenot's criterion was originally developed for applications related to the standard normal distribution (Coleman and Steel 1989). However, as described below, it can be applied to any known or assumed probability density function. The criterion permits rejection of all points that fall outside of a designated band around the sample mean. The width of the band corresponds to an area under the probability density function equal to 1-(1/(2n)) percent, where n equals the number of measurements in the data set. For sample sizes ranging from 5 to 50, the area for the acceptance region ranges from 0.90 to 0.99, corresponding to areas for rejection regions from 0.10 to 0.01, respectively, as shown in Table 3-6. Chauvenot's criterion is always treated as a two-tailed significance test: one-half of the rejection region lies at the upper end of the probability density function and one-half of the rejection region lies at the lower end of the probability density function.

If the distribution of data is assumed to be normal, potential outliers can be compared to the rejection band limits by converting their measured values to a Chauvenot Criterion as follows:

$$\tau_{calculated} = \frac{|y_i - \bar{y}|}{s} \tag{55}$$

where

y_i = the potential outlier

$\bar{y}$ = the sample mean

s = standard deviation for the sample

This conversion is the same as that used for the T-statistic. The values for $\tau_{calculated}$ can then be compared to values for $\tau_{rejection}$, which represent the boundaries of the two-tail rejection region shown in Table 3-5. Maintaining the assumption of normality, the values for $\tau_{rejection}$ can be obtained from any standard normal table, such as that shown in Table A1. Each value for $\tau_{rejection}$ is obtained using the appropriate two-tailed rejection region.

Using the 10 observations from the T-statistic example, determination of $\tau_{rejection}$ would require the use of 0.025 as the two-tailed rejection region, as

Table 3-6
Chauvenot's Criteria for Determination of Outliers (Coleman and Steele 1989)

Number of Samples	Area of Acceptance	Total Area of Rejection	One-Half Rejection Area[1]
5	0.90	0.100	0.050
10	0.95	0.050	0.025
20	0.975	0.025	0.0125
50	0.99	0.010	0.005

[1] To be used for two-tailed tests.

defined in Table 3-6. From Table A1, $\tau_{rejection}$ is found to equal 1.96. The value of $\tau_{calculated}$ for the highest measured asphalt cement content of 5.96 would be 2.33, which is greater than $\tau_{rejection}$. This measurement happens to be the only data point with $\tau_{calculated}$ greater than $\tau_{rejection}$. Similar to the T-statistic conclusion, Chauvenot's criterion indicates that 5.96 should be discarded as an outlier.

Using the data from the Tietjen-Moore (T-M) example, which also included 10 observations, $\tau_{rejection}$ would be defined similar to the T-statistic example (1.96). Using the sample standard deviation of 0.42 percent, the lowest and highest measurements for percent passing would transform to $\tau_{calculated}$ values of 1.17 and 2.40, respectively. The highest measurement (6.9 percent) has a $\tau_{calculated}$ value larger than the $\tau_{rejection}$ of 1.96. This measurement happens to be the only data point with $\tau_{calculated}$ greater than $\tau_{rejection}$. Therefore, similar to the T-M statistic conclusion, Chauvenot's criterion indicates that the measurement of 6.9 percent passing the No. 200 sieve should be discarded as an outlier.

If the distribution is known to conform to a beta shape, rather than normal, the concept of Chauvenot's criterion can still be applied. However, instead of comparing the data to a standard normal distribution, the data is compared to a theoretical beta distribution. The area of the acceptance region can still be calculated as 1-1/(2n) percent and the area of the rejection region as 1/(2n) percent. The rejection region can still be considered two-tailed. The band limits on the distribution, to be used for rejection criteria, can then be determined using Harr's program BETA (Harr 1987). The user of BETA only needs to know the distribution's minimum value, maximum value, mean, and coefficient of variation. If the rejection area has been determined to be 0.10, then the upper band limit corresponds to the value that has only a 5 percent chance of being exceeded by randomly selected values. The lower band limit corresponds to the value that has a 95 percent chance of being exceeded by randomly selected values.

Regardless of the distribution shape or the method of testing for outliers, the test should only be performed one time for a set of data. In other words,

once one or more points have been rejected, the tests for outliers should not be repeated on the new, smaller data set. Repeated application of outlier tests on the same data set would result in an erroneous overall significance level (α).

Components of Variability

When studying the variability of materials, there are two types of analyses that are of primary interest to the construction industry. The first type of analysis is intended to study the variability of a test method or process specifically. In this case, the material to be subjected to the test is collected and homogenized at a single location. Typically, the material is then sent to several laboratories for testing by a single operator from each laboratory. These analyses are often referred to as "round robin" studies because samples from the same homogeneous material are provided to all the parties involved.

The second type of anlysis is intended to differentiate between the variability associated with sampling and testing a heterogeneous material obtained from a construction site, rock quarry, asphalt plant, etc. In order to keep the components of variability reasonable in this type of analysis, a single laboratory should be involved in the testing. These analyses are often referred to as "construction site" studies because the results pertain to a particular project and the results are useful for developing reasonable contract specification criteria.

Round robin studies

Round robin studies are often used for developing precision statements for test methods. In accordance with ASTM (1995a) C 670, two basic elements of precision are needed: single-operator precision and multilaboratory precision. Single operator precision provides an estimate of the difference that may be expected between duplicate measurements made on the same material in the same laboratory by the same operator using the same apparatus within a time span of a few days [ASTM (1995a) C 802]. This within-laboratory precision is often referred to as the repeatability of the test. Multilaboratory precision provides an estimate of the difference that may be expected between measurements made on the same material in two different laboratories [ASTM (1995a) C 802]. This between-laboratory precision is often referred to as the reproducibility of the test method.

At least ten different laboratories should be included in a round robin study [ASTM (1995a) C 802]. The number of different types of a material that should be included in this type of study depend on many factors, such as the range of material types to which the test method should apply, the expense of distributing multiple samples, and the test duration. In general, a minimum of three materials should be included [ASTM (1995a) C 802]. The number of replicates necessary for each test on each material is a function of

the number of laboratories involved. If 15 laboratories are participating, then three replicates is adequate. If less than 15 laboratories are involved, the number of replicates should be increased according to: $(30/p)+1$ where p = the number of laboratories. If more than 15 laboratories are involved, the number of replicates can be reduced to two [ASTM (1995a) C 802].

Example. Assume we are interested in the repeatability and reproducibility associated with determining plasticity index (PI) for soil. The fabricated data for this study, which included 15 laboratories, 3 types of soil, and 3 test replicates, are shown in Table 3-7. Within-laboratory averages and within-laboratory variances are shown in Table 3-8. The overall mean and the pooled variance for PI for each soil type are also shown in Table 3-8.

Table 3-7
Raw Data for a Fabricated Round Robin Experiment Concerning Plasticity Index of Soils

Laboratory	High-Plasticity Soil			Medium-Plasticity Soil			Low-Plasticity Soil		
1	30	31	30	13	11	11	4	3	3
2	32	31	30	10	11	12	8	5	7
3	35	35	34	14	13	14	5	6	5
4	30	30	31	7	6	5	2	4	2
5	36	35	37	11	12	10	5	3	4
6	36	35	36	10	11	11	1	2	3
7	35	36	37	13	11	12	1	2	2
8	31	32	32	12	13	14	6	5	7
9	36	37	38	11	10	11	8	7	6
10	31	30	30	8	8	9	4	5	4
11	31	30	32	11	10	12	5	7	8
12	31	32	32	12	14	13	5	6	7
13	30	31	30	7	6	5	2	4	2
14	32	33	33	11	13	11	3	4	3
15	35	36	36	11	10	11	2	1	3
Overall variance	6.6			5.5			4.1		

Before proceeding with a statistical analysis of the round robin data, we need to verify two assumptions: homogeneity of within-laboratory variances and absence of substantial interactions between laboratory and soil type. If a single laboratory has an erratic variance compared to the others, it may be eliminated. If all the variances are erratic, however, there is a problem with the test method. Substantial interactions between laboratories and materials

Table 3-8
Summary of Mean and Within-Laboratory Variance

Laboratory	High-Plasticity Soil		Medium-Plasticity Soil		Low-Plasticity Soil	
	Mean	Variance	Mean	Variance	Mean	Variance
1	30.3	0.33	11.7	1.33	3.3	0.33
2	31.0	1.00	11.0	1.00	6.7	2.33
3	34.7	0.33	13.7	0.33	5.3	0.33
4	30.3	0.33	6.0	1.00	2.7	1.33
5	36.0	1.00	11.0	1.00	4.0	1.00
6	35.7	0.33	10.7	0.33	2.0	1.00
7	36.0	1.00	12.0	1.00	1.7	0.33
8	31.7	0.33	13.0	1.00	6.0	1.00
9	37.0	1.00	10.7	0.33	7.0	1.00
10	30.3	0.33	8.3	0.33	4.3	0.33
11	31.0	1.00	11.0	1.00	6.7	2.33
12	31.7	0.33	13.0	1.00	6.0	1.00
13	30.3	0.33	6.0	1.00	2.7	1.33
14	32.7	0.33	11.7	1.33	3.3	0.33
15	35.7	0.33	10.7	0.33	2.0	1.00
Pooled[1,2]	33.0	0.55	10.7	0.82	4.2	1.00

[1] Overall mean [= ($\sum$within-lab means)/number of labs].
[2] Pooled within-laboratory variance [= ($\sum$within-lab variances)/number of labs].

would also indicate either a problem with a laboratory or a problem with the test method. In an extreme case of interaction, the laboratories may not report the same ranking of materials, in terms of the magnitude of test results.

Homogeneity of variances for each material may be tested with the Hartley F-max test (Freund and Wilson 1993). This method simply involves finding the largest ratio of within-laboratory variances and then comparing this ratio to the F-max distribution data shown in Table A7 (Appendix A). The null hypothesis for this test states that the variances are homogeneous. The alternative hypothesis states that at least two variances are not equal. If the calculated ratio is larger than the critical value shown in Table A7, then the null hypothesis would be rejected. The largest ratio of variances found within high-plasticity soil, medium-plasticity soil, and low-plasticity soil was 3.0 (1.0/0.33), 4.0 (1.33/0.33), and 7.0 (2.33/0.33), respectively. For 15 laboratories, 3 replicates per laboratory, and a level of significance of 5 percent, the critical F-max value is 855. The critical F-max value is high

because our estimates for within-laboratory variability are poor (only 3 replicates). We have no reason to reject the null hypothesis of equal variances.

Substantial interactions can be checked by inspecting the reported means for plasticity index in Table 3-8. If we ranked the reported means from each laboratory, we would find that each laboratory reported the lowest PI for the same material and the highest PI for the same material. There does not seem to be any laboratory that reported a different trend in results, relative to the other laboratories. We can state that there are no substantial interactions.

Having verified the assumptions of homogeneous variances and the absence of substantial interaction, a statistical computer program such as SigmaStat® or SAS® can be used to perform an analysis of variance (ANOVA) procedure for each soil type. This permits estimation of the between-laboratory variances and verification of the pooled within-laboratory variances. Since the number of replicates was the same for each laboratory, the pooled within-laboratory variance in this case is calculated as a simple average. The analysis is similar to a completely randomized experimental design where laboratories are the treatment factors and the variation within laboratories reflects experimental error and sampling error. Summary ANOVA output tables are shown in Tables 3-9 through 3-11.

Table 3-9
Round Robin Analysis of Variance for Plasticity Index Using High-Plasticity Soil

Source of Variability	Degrees of Freedom[1]	Sum of Squares	Mean Square	Expected Mean Square[2]
Laboratory	$p-1 = 14$	273.2	19.52	$\sigma_w^2 + r(\sigma_b^2)$
Error	$p(r-1) = 30$	16.7	0.55	σ_w^2
Corrected total	$pr-1 = 44$	289.9	N/A	N/A

[1] p = Number of labs, r = number of tests per lab.
[2] σ_b^2 = Variance between labs, σ_w^2 = variance within labs.
N/A No calculation.

The ANOVA tables list two individual sources of variability: error and laboratory. The error component represents within-laboratory variability. The laboratory component includes both within-laboratory variability and between-laboratory variability. The ANOVA results include the calculation of a sum of squares term and a mean square term for each source of variability, as shown. A sum of squares is simply the sum of squared deviations from the mean, as discussed in Chapter 2. A mean square is the sum of squares divided by the degrees of freedom for the particular source of variability. The mean square terms can be compared to the expected mean square formulas, which are known because they comprise the mathematical basis of the ANOVA. Collectively, these comparisons provide two equations and two unknowns (within-laboratory variance and between-laboratory

variance). After calculating these variances, they can be added to estimate the overall variance. These computed results are summarized in Table 3-12.

Table 3-10
Round Robin Analysis of Variance for Plasticity Index Using Medium-Plasticity Soil

Source of Variability	Degrees of Freedom[1]	Sum of Squares	Mean Square	Expected Mean Square[2]
Laboratory	p-1 = 14	219.0	15.64	$\sigma_w^2 + r(\sigma_b^2)$
Error	p(r-1) = 30	24.7	0.82	σ_w^2
Corrected total	pr-1 = 44	243.6	N/A	N/A

[1] p = Number of labs, r = number of tests per lab.
[2] σ_b^2 = Variance between labs, σ_w^2 = variance within labs.
N/A No calculation.

Table 3-11
Round Robin Analysis of Variance for Plasticity Index Using Low-Plasticity Soil

Source of Variability	Degrees of Freedom[1]	Sum of Squares	Mean Square	Expected Mean Square[2]
Laboratory	p-1 = 14	150.3	10.74	$\sigma_w^2 + r(\sigma_b^2)$
Error	p(r-1) = 30	30.0	1.00	σ_w^2
Corrected total	pr-1 = 44	180.3		

[1] p = Number of labs, r = number of tests per lab.
[2] σ_b^2 = Variance between labs, σ_w^2 = variance within labs.

Table 3-12
Summary of Round Robin PI Data by Soil Type

Soil Type	Mean, Percent	Components of Variance		
		Within-Laboratory	Between-Laboratory	Overall Variance[1]
Low-plasticity	4.2	1.00	3.25	4.25
Medium-plasticity	10.7	0.82	4.94	5.76
High-plasticity	33.0	0.55	6.32	6.87

[1] Overall variance = within-laboratory variance + between-laboratory variance.

A comparison between Tables 3-8 and 3-12 reveals that the ANOVA-derived within-laboratory variances match the hand-calculated pooled variances exactly. Inspection of Table 3-12 also reveals that the between-laboratory variances were approximately three to eleven times larger than the within-laboratory variances. The estimated variances and the calculated means can be used to calculate standard deviations and coefficients of variation, as shown in Table 3-13. The soil types have been organized in the table in order of increasing mean to permit a search for trends in standard deviation and coefficient of variation. These trends will determine the most appropriate form for the precision statements.

Table 3-13
Standard Deviations and Coefficients of Variation for PI Data

Soil Type	Mean, Percent	Standard Deviation, Percent		Coefficient of Variation, Percent	
		Within-Laboratory	Between-Laboratory	Within-Laboratory	Between-Laboratory
Low-plasticity	4.2	1.00	1.80	24	43
Medium-plasticity	10.7	0.91	2.22	8.5	21
High-plasticity	33.0	0.74	2.51	2.2	7.6

The form of precision statements will depend on the relationship between standard deviation (within-laboratory and between-laboratory) and mean material property. Generally, this relationship will fall into one of two categories: either the standard deviation remains approximately constant over the range of mean values or the coefficient of variation remains approximately constant over the range of mean values. In the case of constant standard deviation, the coefficient of variation would have to decrease as mean increases. In the case of constant coefficient of variation, the standard deviation would have to increase as mean increases. Inspection of Table 3-13 reveals that for this round robin experiment, standard deviation remained approximately constant across the range of mean PI, for both within-laboratory and between-laboratory components of variability. The precision statement would therefore be most useful if written in terms of standard deviation, rather than in terms of coefficient of variation.

The precision statement developed from this fabricated round robin experiment is shown in Table 3-14. The standard deviations reported in Table 3-14 were calculated by averaging over the three soil types included in the study. Assuming that the test method specification will require two replicates, the precision statement will include an allowable difference for two measurements, which is calculated by multiplying the standard deviation by 2.8. According to ASTM (1995a) C 670, this difference between two measurements represents that which should be exceeded, on the average, only 5 percent of the time. There are many other forms of precision statements, including those that are based on coefficient of variation. Also,

Table 3-14
Precision Statement for Plasticity Index

Type Index	Standard Deviation, Percent	Acceptable Range of Two Test Results, Percent
Single-operator	0.88	2.5[1]
Multilaboratory	2.18	6.1[2]

[1] Each test result is one of two replicates from a single laboratory.
[2] Each test result is a single replicate and each replicate is from a different laboratory.

precision statements can be written for specifications that require more than two replicates. Detailed information on developing precision statements is provided in ASTM (1995a) C 670 and ASTM (1995b) E 177.

Construction site studies

The variabilities of concern for construction site operations typically involve a single testing laboratory. Multiple laboratories would not normally be involved in a single project's quality control or quality assurance. Typically, a contractors' performance and subsequent payment are judged for each "lot" of material separately. A lot can be defined as an isolated quantity of material or process accumulated under conditions that are considered uniform for sampling purposes [ASTM (1995b) E 105, AASHTO (1995) R 10]. Examples of lots include 1,800 metric tons of asphalt concrete, 8 hrs worth of production of asphalt concrete, a single lift of select fill considered to be constructed with a uniform material and under uniform conditions, or a 1,000-ft section of completed airfield taxiway.

The concepts presented in the following discussion are applicable for sampling and testing all construction materials, but portland cement concrete will be used as the example. A single lot of concrete can consist of many truckloads, each of which can be considered a "batch." The overall variation of concrete in a lot can therefore be broken down into between-batch variation and within-batch variation. Between-batch variation represents the difference in test results from one batch to another for a material that is supposedly produced consistently.

The within-batch variation, which represents the variabilities that exist within any single batch, can be broken down into sampling error and experimental error. Sampling error is caused by some inconsistency in the sampling procedure that disturbs the random selection process. A popular example is sampling aggregate from a segregated stockpile. Experimental error can be further broken down into two components: testing error and inherent variability. Testing errors are caused by inconsistencies that occur while performing the same test on multiple samples. Testing variation is affected by the repeatability of a test, technician skills, the condition of equipment, and the methods used for reducing samples to a size that is

appropriate for testing (Nicotera 1974). Testing variation can be quantified by repeating the same test on similar samples of material. Inherent variability is governed by the laws of chance and it is unavoidable. Its magnitude will vary depending on the material and the test. For a particular situation, the influence of inherent variability on an analysis of variance (ANOVA) can only be reduced by increasing the number of test replicates (Waller 1966).

Between-batch variation typically exceeds within-batch variation. This relationship is similar to the case where between-laboratory variation typically exceeds within-laboratory variation for round robin experiments (Nicotera 1974).

The components of variance for a construction site material study are additive. The Pythagorean theorem provides a visual tool for relating variance components back to standard deviations (Newlon 1966, Waller 1966). If the length of each orthogonal side of a right triangle is equal to a component standard deviation, the variance associated with the hypotenuse equals the sum of the orthogonal sides squared, as shown in Figure 3-4. Overall variance equals between-batch variance plus within-batch variance. Within-batch variance equals sampling variance plus variance associated with experimental error. Experimental variance is composed of testing variability and inherent material variability, but these components are difficult to separate (Nicotera 1974).

$$s^2_{overall} = s^2_{between-batch} + s^2_{within-batch} \tag{56}$$

$$s^2_{within-batch} = s^2_{sampling} + s^2_{experimental} \tag{57}$$

Between-batch variance reflects the "control" of a construction operation, so it is typically the quantity needed for judging contractor performance. If the batch-to-batch variance is large relative to the within-batch variance, the operation can be judged "out of control" (Waller 1966). Accurate determination of material variances is facilitated by two procedures: (1) reducing sampling and testing variabilities through standardization of sampling and testing procedures and (2) increasing the number of test replicates within a sampling plan (Newlon 1966).

Similar to the round robin experiment, different components of variance can be isolated with an anlaysis of variance (ANOVA) procedure. The procedure used for the ANOVA will depend on the number and nature of the different sources of variation.

Example. Brown (1966) used an analysis of variance procedure to isolate sources of variation for the compressive strengths of steam cured concrete. He was concerned with the relative proportions of variation that existed between concrete batches and within concrete batches: $\sigma^2_{total} = \sigma^2_{between} + \sigma^2_{within}$. A large variation between groups, relative to the variation within

groups, would indicate the existence of a material control problem. Example data is shown in Table 3-15.

Table 3-15
Compressive Strengths of Steam-Cured Concrete, MPa (after Brown 1966)

Batch No.	Sample No. 1	Sample No. 2	Total
1	30.3	30.1	60.4
2	31.2	31.4	62.6
3	28.1	29.0	57.1
4	31.7	32.3	64.0
5	31.2	31.7	62.9
Overall total (for N = 10 samples)			307.0
Overall average			30.7

Due to the few sources of variation and the small dataset, Brown (1966) was able to demonstrate the ANOVA with hand calculations, as shown in Table 3-16. This analysis is similar to a completely randomized experimental design where concrete batches are the treatment factors and variation within batches reflects experimental error and sampling. The between-batch component of variance accounted for a significant proportion (93 percent) of the overall variance.

If this ANOVA had been performed with commercial software, such as SAS®, the output would look similar to Table 3-17. Similar to the ANOVA in the round robin experiment, the calculated mean squares can be compared to the expected mean squares to obtain estimates for two sources of variability. In this case, the two sources are within-batch variance and between-batch variance. Results from calculations using the ANOVA output are summarized in Table 3-18. Similar to the hand calculations, the between-batch variance accounted for 92 percent of the total variance. The large ratio of between-batch variance to within-batch variance indicated to Brown (1966) that he had a concrete production control problem.

Example. In a study with a similar experimental design, Hughes and Anday (1966) used analysis of variance techniques to optimize the efficiency of nuclear density gage quality control testing. In order to optimize efficiency, they needed to differentiate between two components of variance within a typical lot of soil: between-test variability and between-test-site variability. Between-test variability referred to variance between repeated tests performed at the same location. Between-test-site variability referred to variance between the test results at different locations within the same lot.

Table 3-16
Example of Computational Procedures for Variance Analysis (after Brown 1966)

Computational Steps	Sources of Variation: Between-Batches	Within-Batches	Total
a. Enter squared values	$60.4^2 + 62.6^2 + \ldots + 62.9^2$	N/A	$30.3^2 + 30.1^2 + \ldots + 31.7^2$
b. Sum of "a"	18,879.7	N/A	9,440.6
c. No. of tests per each entry in "a"	2	N/A	1
d. Crude sum of squares, "b"/"c"	9439.9	N/A	9440.6
e. Correction factor, C = (grand total)^2/N	9424.9	N/A	9424.9
f. Corrected sum of squares	(d_1^a - e) 15.0	(f_3 - f_1) 0.7	(d_3 - e) 15.7
g. Degrees of freedom	(# batch - 1) 4	(g_3 - g_1) 5	(N - 1) 9
h. Mean square, "f"/"g"	3.75	0.14	N/A
I. Component of variance	(h_1 - h_2)/c_1 1.81	h_2 0.14	(i_1 + i_2) 1.95
j. Component of variance, %	93	7	100

N/A No calculation.
[a] Subscripts represent column numbers under "Sources of Variation."

Table 3-17
Analysis of Variance for Concrete Compressive Strength

Source of Variability	Degrees of Freedom[1]	Sum of Squares	Mean Square	Expected Mean Square[2]
Batches	p-1 = 4	14.97	3.74	$\sigma_w^2 + r(\sigma_b^2)$
Error	p(r-1) = 5	0.75	0.15	σ_w^2
Corrected total	pr-1 = 9	15.72	N/A	N/A

[1] p = Number of batches, r = number of tests per batch.
[2] σ_b^2 = Variance between batches, σ_w^2 = variance within batches.
N/A No calculation.

They selected five sections of roadway, each with a different type of soil, and they analyzed the variance components for each section separately. Eight sites for testing were selected randomly within each section and two tests (nuclear gage determinations) were performed at each site. The

Table 3-18
Summary of ANOVA Calculation Results for Concrete Compressive Strength

Soil Type	Components of Variance		
	Within-Laboratory	Between-Laboratory	Overall Variance[1]
Component of variance (MPa^2)	0.15	1.80	1.95
Percent of variance	8	92	100

[1] Overall variance = within-laboratory variance + between-laboratory variance.

between-test-site variation provided an indication of the variabilities associated with the material and the construction process. The between-test variation provided an indication of the repeatability of the nuclear gage apparatus when used on the same material. Hughes and Anday (1966) found that the variation between tests accounted for a very small proportion of the total variation. The variation between test-site means, however, was statistically significant. They concluded that if sixteen tests were to be performed within a section (or lot), the standard error for section means would be minimized by selecting sixteen different sites and running only one test per site. Statisticians refer to this as maximizing experimental precision (Petersen 1985).

Hughes and Anday (1966) were not concerned with variance caused by sampling procedures because the repeated tests were performed in the same access hole. In the concrete production example, Brown (1966) also did not feel it necessary to differentiate between the components of within-batch variation. In some cases, however, an engineer may want to break down within-batch variation into sampling error and experimental error. These types of analyses become slightly more complex.

In order to differentiate between sampling variance and experimental variance, one needs a sampling plan such as that shown in Figure 3-5. Multiple batches are randomly selected from a lot of material and then at least two samples are randomly selected from each batch. Finally, material for at least two individual test replicates are split out of each sample. The number of batches selected from each lot would be decided as part of a construction specification acceptance plan. This decision is based on a number of factors, including predetermined party risks, anticipated material variabilities, and type of payment adjustment scheme. It is important to "split out" the test replicates from the samples. The analysis must be able to assume that the test replicates are similar representations of the same sample.

The ANOVA compares the results of individual test portions to define an experimental (testing and inherent) variation. By mathematically combining the test portions within each sample, the ANOVA can compare samples to estimate the sampling variation. Each set of sample increments is then

mathematically combined into an average batch value to permit determination of batch-to-batch variation (Nicotera 1974).

Example. Data presented by Nicotera (1974) will be used to demonstrate this type of anlysis of variance. The data in Table 3-19 are values for percent passing the 19-mm sieve for aggregates obtained from asphalt concrete extractions. Two samples (A and B) were obtained from 40 different batches. Each sample had enough material for two extraction/gradation tests.

Hand calculations for this level of ANOVA would be tedious. Results provided by SAS® software, however, are shown in Table 3-20. The experimental design can be considered as a completely randomized treatment design with nested subsampling. The batches are the treatment factors and the samples are nested within batches. The variability within samples represents experimental error. Similar to the previous examples, components of variance can be estimated using the expected mean square formulas and the calculated mean squares shown in Table 3-20. Results for these calculations are shown in Table 3-21. Inspection of the components of variance reveals that variation between batches accounted for 36 percent of the overall variance. Testing and inherent variation accounted for 45 percent of the overall variance, which is a relatively large proportion. Sampling accounted for only 19 percent of the overall variance. At first glance, the high proportion of variance within batches (45+19=64 percent) would indicate that the procedure for measuring percent passing the 19-mm sieve was not very consistent. However, upon more thorough study, this high proportion of the variance is attributable to a low overall variance. The overall coefficient of variation for percent passing the 19-mm sieve was only 2.7 percent. The control of coarse aggregates for the production of asphalt concrete appears to be in control. The repeatability of asphalt extraction/gradation tests appears to be favorable.

As a method for checking the ANOVA results, the overall variance can be calculated by spreadsheet and compared with the estimated value shown in Table 3-21. Overall variance is simple to calculate because it includes all the data (160 values), without regard for batch, sample, or test designations. Overall variance in this case was calculated to be 6.08 percent^2, which agreed with the ANOVA results.

Table 3-19
Percent Passing the 19-mm Sieve (adapted from Nicotera 1974)

Batch	A1	A2	B1	B2
1	88.4	89.0	89.4	87.8
2	88.8	90.1	91.4	90.4
3	90.0	90.7	93.0	91.7
4	91.2	90.6	89.0	95.7
5	90.2	91.9	89.0	90.0
6	92.0	91.4	90.4	90.0
7	88.1	89.5	90.4	88.4
8	90.6	91.4	89.6	89.8
9	89.3	89.2	94.9	90.7
10	95.4	91.5	88.7	89.4
11	90.8	92.6	92.0	91.2
12	94.3	92.7	86.4	86.7
13	89.5	87.2	88.4	87.7
14	88.2	88.5	90.4	90.6
15	85.5	87.6	87.4	87.1
16	88.9	87.4	88.6	89.5
17	94.0	89.7	87.2	87.5
18	90.1	89.0	85.6	90.3
19	92.6	92.0	89.9	91.2
20	89.5	87.2	87.8	88.3
21	86.6	88.4	84.3	87.2
22	87.6	88.8	90.7	84.9
23	89.2	87.6	91.2	90.7
24	85.8	87.9	89.2	91.0
25	89.2	92.3	87.6	95.7
26	90.7	89.7	93.8	92.2
27	90.4	85.3	90.2	90.5
28	92.2	90.9	90.1	90.6
29	90.5	91.1	90.5	90.2
30	92.2	91.4	88.8	88.7
31	93.0	92.9	92.0	93.0

(Continued)

Table 3-19 (Concluded)				
Batch	**A1**	**A2**	**B1**	**B2**
32	91.0	90.5	87.4	91.1
33	90.5	87.0	87.0	82.8
34	87.0	87.1	86.2	87.2
35	92.2	91.3	89.8	91.2
36	88.5	90.9	91.2	92.6
37	91.6	91.5	91.4	91.6
38	93.4	94.3	93.4	96.0
39	84.3	89.4	87.0	89.3
40	96.0	93.6	92.4	95.2

Table 3-20
Analysis of Variance for Percent Passing the 3/4-in. Sieve

Source of Variability	Degrees of Freedom[1]	Sum of Squares	Mean Square	Expected Mean Square[2]
Batch	p-1 = 39	536.62	13.760	$\sigma_t^2 + s(\sigma_s^2) + rs(\sigma_b^2)$
Sample (batch)	p(r-1) = 40	200.52	5.013	$\sigma_t^2 + s(\sigma_s^2)$
Error	pr(s-1) = 80	220.44	2.756	σ_t^2
Corrected total	prs-1 = 159	957.58	N/A	N/A

[1] p = Number of batches, r = number of samples per batch.
s = Number of tests per sample.
[2] σ_b^2 = Variance between batches, σ_s^2 = variance attributable to sampling.
σ_t^2 = Variance attributable to testing plus inherent variability.
N/A No calculation.

Table 3-21
Components of Variance for Percent Passing the 19-mm Sieve

Source of Variability	Variance in Units of Percent Squared	Component Percent of Total Variance
Between batches	2.19	36
Experimental (testing and inherent)	2.76	45
Sampling	1.13	19
Within batches	2.76 + 1.13 = 3.89	45 + 19 = 64
Overall	3.89 + 2.19 = 6.08	64 + 36 = 100

Overall coefficient of variation = $(\sqrt{6.08}/90)*100$ percent = 2.7 percent.

4 Historical Variability Data

As part of this study, a literature review was performed to collect variability data related to pavement materials and pavement structures. These data are necessary for the proper development of specification criteria, including pay adjustment factors. Details concerning the data found in literature, including references, are presented in Appendices B through F. This chapter serves as a summary for the appendices.

The material variabilities reported in this chapter involved a single laboratory. Variabilities associated with multiple laboratories would typically not be an issue for project quality testing. Since contractor pay is typically calculated for individual lots on a paving project, the variabilities reported in this chapter are intended to reflect total within-lot variabilities. Total within-lot variabilities include between-batch variabilities, between-sample variabilities, and experimental variation (testing and inherent).

When several estimates for variability were provided by the same source, variances were "pooled" into a single estimate as described in Chapter 2. If the number of replicates for variance estimates were not given, pooling of variances was performed as a straight average (replicates were assumed to all be equal).

Material variabilities are summarized in this chapter with both reported standard deviations (SDs) and coefficients of variation (CVs). Reported variabilities for any particular material property followed one of three trends with respect to the property mean:

a. As the mean increased, the standard deviation remained approximately constant while the coefficient of variation decreased.

b. As the mean increased, the coefficient of variation remained approximately constant while the standard deviation increased.

c. The range of reported means was too small to decipher these trends.

If SD remained approximately constant, SD is summarized as a single value and CV is summarized as a range. If CV remained approximately constant, CV is summarized as a single value and SD is summarized as a range. If the

range of reported means was too small to decipher these trends, SD and CV are both summarized as single values.

Any comments found in literature concerning the normality of data are included in this chapter. In most cases, these comments are just perceptions related to the shape of frequency distributions. However, in some cases, a normality test was performed. All reports for normality address individual test results. This chapter does not address any distributions for sample means.

Residual Fine-Grained Soil Deposits

Krahn and Fredlund (1983) reported on several soil deposits in Canada. They noted a surprising consistency with respect to the variability of individual soil properties between deposits. This finding supports the logic behind collecting and summarizing variability data for use in future construction projects.

The measured properties within a "lot" (uniform deposit) of residual soil typically exhibit coefficients of variation in the range of 10 to 30 percent (Table 4-1). Measured properties with relatively low variability include specific gravity of soil particles and dry density. Measured properties with relatively high variabilities include permeability, plasticity index, unconfined compressive strength, and shear box strength results. When soil strength tests require field sampling and transportation of these samples to a laboratory, the variability of measured strength is highly dependent on sampling technique (Kay and Krizek 1972).

Some of the measured properties were reported to be non-normal: natural water content (percent), degree of saturation (percent), maximum dry density (kg/m^3), and unconfined compressive strength (kPa). Two of these properties can most likely attribute their deviations from normality to limiting numerical boundaries. Natural water contents (w) cannot fall below 0 percent, so the distribution for a soil with a low mean w would tend to exhibit a positive skew. Degree of saturation (S) cannot exceed 100 percent, so the distribution for a soil with a high mean S would tend to exhibit a negative skew. Unconfined compressive strength can most likely attribute its deviation from normality to heterogeneity induced by sampling. Sampling would have the potential to decrease measured strength, causing the distribution to exhibit a negative skew.

Engineered Fill

The measured properties within a "lot" of engineered fill typically exhibit coefficients of variation in the range of 10 to 30 percent (Tables 4-2 and 4-3). Relative compaction had a low variability (approximately 5 percent), possibly because it is often used for payment. Contractors typically have

Table 4-1
Summary of Reported Variabilities for Residual Fine-Grained Soil Deposits

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Specific gravity	N/A	2	normal[1]
Natural water content, percent	4	10-25	conflicts[2]
Porosity	0.05	10-20	normal[1]
Void ratio	0.15	20	normal[1]
Degree of saturation, percent	8	7-15	not normal[1]
Dry density, kg/m^3	105	5	not reported[3]
Coefficient of permeability, percent			not reported[3]
- 80 percent saturated	N/A	90	
- 100 percent saturated	N/A	700	
Liquid limit	4-15	20	normal[1]
Plastic limit	2-5	15	not reported[3]
Plasticity index	3-10	40	not reported[3]
Particles finer, percent			not reported[3]
- No. 40 sieve	15	15	
- No. 200 sieve	10	20	
- 2 μm sieve	10	35	
Compression index	0.030-0.070	30	not reported[3]
Swell index	0.020	25-55	not reported[3]
Optimum moisture Content, percent	5	20	normal[4]
Maximum dry density, kg/m^3	130	8	not normal[4]

(Continued)

[1] According to the chi-square "goodness-of-fit" test.
[2] Reports concerning normality conflicted.
[3] Reports on variability did not address normality.
[4] Based on the shapes of histograms.
N/A Limited data or not reported.

Table 4-1 (Concluded)

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Unconfined compression strength, kPa	7-190	45	not normal[1]
California bearing ratio, percent	3	25	not reported[3]
Field vane, kPa	8	30	not reported[3]
Undrained triax. Test - c - tanϕ	 N/A N/A	 25 20	not reported[3]
Drained triaxial test - c - tanϕ	 N/A N/A	 15 2	not reported[3]
Drained shear box - c - tanϕ	 N/A N/A	 100 30	not reported[3]

[1] According to the chi-square "goodness-of-fit" test.
[3] Reports on variability did not address normality.
N/A Limited data or not reported.

Table 4-2
Summary of Reported Variabilities for Compacted Fine-Grained Soil Deposits

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Controlled moisture content, percent	2	20	conflicts[1]
Relative compaction, percent	4	5	conflicts[1]
Resilient modulus, MPa	25	25	not reported[2]
California bearing ratio, percent	1-15	30	not reported[2]
Modulus of subgrade reaction, MPa/mm	N/A	35	not reported[2]
Benkelman beam deflections, mm	0.35	25	not reported[2]
Dynaflect tests, MPa	N/A	35	not reported[2]

[1] Reports concerning normality conflicted.
[2] Reports on variability did not address normality.
N/A Limited data or not reported.

Table 4-3
Summary of Reported Variabilities for Lime-Soil Mixtures and Soil-Cement

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Unconfined compression strength, Mpa	N/A	10	not reported[1]
Splitting-tensile strength, MPa	N/A	10	not reported[1]
Flexural strength, MPa	N/A	10	not reported[1]

[1] Reports on variability did not address normality.
N/A Limited data or not reported.

financial interest in meeting density requirements and this objective is facilitated by keeping density variabilities as low as possible. The CV for both controlled moisture content and California bearing ratio are in the same range as the corresponding values for residual soil. The CV for unconfined compressive strength, however, was lower for engineered fill than for residual soil.

Inspection of Tables 4-2 and 4-3 also reveals that conflicts existed for some of the reports on normality. These conflicts were present for both controlled moisture content (percent) and relative compaction (percent). The tendency for these distributions to be skewed may be affected by mean value and variability. Controlled moisture content would tend to exhibit a normal distribution when the mean is far removed from the numerical limit of zero. Relative compaction would tend to exhibit a normal distribution when the variability is low. A high variability for relative compaction with a mean value close to 100 percent would tend to induce a negative skew.

Subbase and Base Course Materials

The liquid limit and plasticity index for untreated subbase and base course materials have SDs that are approximately the same as for the fine-grained soils. The lower mean values for these properties for subbase and base courses, however, cause the CVs to be higher than for the fine-grained soil (Table 4-4). Similar to fine-grained soil, permeability measurements for subbase and base course materials are highly variable. Standard deviations (SDs) for percent passing individual sieve sizes range from approximately 1 percent to 5 percent, while the CVs for percent passing individual sieve sizes range from approximately 5 percent to 50 percent. Measurement with fine sieves tend to have the highest CVs because they have the lowest mean values for percent passing. Finer sieve sizes also tend to be susceptible to errors induced by sampling method, screen cleanliness, and skill of the

Table 4-4
Summary of Reported Variabilities for Subbase and Base Course Materials

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Controlled moisture content, percent	1	10	not reported[1]
Liquid limit, percent	5	10-80	not reported[1]
Plasticity index, percent	2	25-65	not normal[2]
Coefficient of permeability, percent - silty sand	N/A	240	not reported[1]
Particles finer, percent - 19-mm sieve - 13.2-mm sieve - 9.5-mm sieve - No. 4 sieve - No. 8 sieve - No. 16 sieve - No. 30 sieve - No. 50 sieve - No. 100 sieve - No. 200 sieve	 3 2 5 5 3 4 3 3 2 1	 3-10 3 3-20 4-20 5-15 20-25 10 5-25 30-45 5-25	not reported[1]
Sand equivalence, percent	5	15	not reported[1]
Magnesium sulfate soundness, percent	0.20-1.5	3	not reported[1]
Lift thickness, mm - subbase - base	 8-55 5-30	 35 15	not reported[1]
Relative compaction, percent - subbase - base	 3 2	 3 2	not reported[1]
Angle of internal friction, ϕ	N/A	10	not reported[1]
Unconfined compression strength, kPa	N/A	10	not reported[1]
California bearing ratio (field), percent	8-35	25	not reported[1]
Benkelman beam deflections, mm	0.25	20	not reported[1]

[1] Reports on variability did not address normality.
[2] Based on the shapes of histograms.
N/A Limited data or not reported.

technician (Michigan State Department of Highways 1966). Similar to the fine-grained soils, relative compaction had a low variability.

Tests for mechanical properties for subbase and base course materials tend to have high variabilities, as shown in Tables 4-5 and 4-6. Laboratory tests for modulus, Poisson's ratio, and fatigue properties had particularly high variabilities. The plate-load test, which is performed in the field, also had a particularly high variability.

Table 4-5
Summary of Reported Variabilities for Lime- and Cement-Stabilized Subbase and Base Course Materials

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Unconfined compression strength, kPa	840	15	not reported[1]
Compression modulus, MPa	2,400	60	not reported[1]
Indirect tensile strength, kPa	340	35	normal[2]
Tensile modulus, MPa	420	70	not reported[1]
California bearing ratio (field), percent	90	30	not reported[1]
Plate-load tests, MPa/mm	N/A	70	not reported[1]
Dynaflect tests, MPa	N/A	20	not reported[1]

[1] Reports on variability did not address normality.
[2] Based on the shapes of histograms.
N/A Limited data or not reported.

Table 4-6
Summary of Reported Variabilities for Asphalt-Stabilized Base Course Materials

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Indirect tensile strength, kPa	150	20	normal[1]
Tensile modulus, MPa	140	35	not reported[2]
Indirect poisson's ratio	0.15	50	not reported[2]
Flexural stiffness, Mpa	130-1,090	25	not reported[2]
Fatigue life of beams	N/A	50	not reported[2]

[1] Based on the shapes of histograms.
[2] Reports on variability did not address normality.
N/A Limited data or not reported.

Asphalt Concrete

Penetration measurements for asphalt cement had lower CVs than capillary viscosity measurements, as shown in Table 4-7.

Standard deviations for percent passing individual sieve sizes for asphalt concrete aggregates, as shown in Table 4-8, were very similar to those for subbase and base course materials. The CVs for bulk specific gravity, maximum theoretical gravity, and field density were all relatively low. The CV for voids total mix was high due to the small mean values (typically 3 to 6). Similar to stabilized subbase and base course materials, measurements of modulus, Poisson's ratio, and fatigue had high variabilities.

For field tests, deflections measured by a falling-weight deflectometer (FWD) were generally less variable than deflections measured by a Benkelman beam (Table 4-9). Backcalculated moduli for FWD were least variable for the subgrade and most variable for asphalt concrete.

Table 4-7
Summary of Reported Variabilities for Asphalt Cement

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Penetration at 25°C, 0.01 mm	2-20	25	not reported[1]
Viscosity at 60°C, Pa-s	240-1270	55	not reported[1]

[1] Reports on variability did not address normality.

Portland Cement Concrete

The SDs for percent passing large aggregate sieve sizes is typically larger for portland cement concrete (PCC) than for asphalt concrete (AC), as shown in Table 4-10. This trend may be a reflection of the less strict aggregate control requirements for PCC construction, as compared to AC construction. Ready-mix PCC plants typically include only two aggregate feed bins, while asphalt plants typically include four bins.

The average reported CV for compressive strength was 15 percent, as shown in Table 4-11. This value reflects favorable quality control according to Baker and McMahon (1969). In cases of poor quality control, CV for compressive strength exceeds 25 percent. The average reported CV for flexural strength was 7 percent, which reflects favorable quality control according to Witczak et al. (1983). In cases of poor quality control, CV for flexural strength exceeds 20 percent.

Table 4-8
Summary of Reported Variabilities for Asphalt Concrete Materials

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Mixture temperature in the field, °C	8	6	not reported[1]
Extracted asphalt cement content, percent	0.25	4	not reported[1]
Particles finer, percent			not reported[1]
- 25-mm sieve	5	5	
- 19-mm sieve	2	2	
- 13.2-mm sieve	2	2	
- 9.5-mm sieve	2	2-3	
- No. 4 sieve	4	1-10	
- No. 8 sieve	3	5-10	
- No. 16 sieve	3	5-10	
- No. 30 sieve	2	5-10	
- No. 50 sieve	2	10	
- No. 100 sieve	1	10	
- No. 200 sieve	1	10-20	
Bulk specific gravity	0.030	1	not reported[1]
Theoretical maximum specific gravity	0.010	1	not reported[1]
Voids total mix, percent	1.0	10-65	not reported[1]
Voids filled, percent	5	6	not reported[1]
Field density, percent relative to laboratory	1.4	2	not reported[1]
Marshall stability, kN	1.3	15	not reported[1]
Marshall flow, mm	0.30	15	not reported[1]
Indirect tensile strength, kPa	85	15	not reported[1]
Indirect static modulus, MPa	80-480	55	not reported[1]
Indirect poissons ratio	N/A	40	not reported[1]
Dynamic modulus, MPa			not reported[1]
- 4°C	N/A	15	
- 21°C	N/A	30	
- 38°C	N/A	20	
Flexural stiffness, MPa			not reported[1]
- 4°C	520-1020	20	
- 21°C	150-280	20	
Fatigue life at 21°C, cycles	N/A	65	not reported[1]

[1] Reports on variability did not address normality.
N/A Limited data or not reported.

Table 4-9
Summary of Reported Variabilities for Field Measurements on Asphalt Pavements

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Mat thickness, mm	8	10	not reported[1]
Benkelman beam deflections, mm	2	15-70	not normal[2]
Falling weight deflectometer (FWD) Deflections			normal[2]
- under load, μm	30-340	15	
- at 0.9 m offset, μm	8-30	15	
Falling weight deflectometer (FWD) Modulus calculations			not normal[2]
- AC modulus, MPa	210-3450	50	
- base modulus, MPa	20-240	40	
- subgrade modulus, MPa	2-115	15	
Initial serviceability index	0.35	8	normal[3]

[1] Reports on variability did not address normality.
[2] Based on the shapes of histograms.
[3] According to the chi-square "goodness-of-fit" test.

Coefficients of variation for load transfer ranged from 20 to 55 percent. Tied contraction joints had the lowest variability. Keyed construction joints and doweled contraction joints had the highest variabilities. Variabilities in falling weight deflectometer (FWD) deflections and FWD moduli were similar to those for flexible pavements.

Frequency distributions for most of the fresh and hardened concrete properties have been reported to be normal in shape, as shown in Tables 4-10 and 4-11. The measured air content of air-entrained fresh concrete was an exception; its distribution was reported to have a negative skew. The FWD moduli were also an exception; in some cases they exhibited positive skew.

Table 4-10
Summary of Reported Variabilities for Portland Cement Concrete Materials

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Air content, percent	0.85	15	not normal[1]
Slump, mm	8-25	30	normal[1]
Particles finer, percent (coarse aggregates)			normal[1]
- 25-mm sieve	3	2-4	
- 19-mm sieve	10	4-20	
- 13.2-mm sieve	10	15-35	
- No. 4 sieve	1	45-90	
- No. 8 sieve	1	65	
Particles finer, percent (fine aggregates)			normal[1]
- No. 4 sieve	1	1	
- No. 8 sieve	3	3	
- No. 16 sieve	4	6	
- No. 30 sieve	4	9	
- No. 50 sieve	5	15-40	
- No. 100 sieve	1	20-60	
Fineness modulus	0.10	4	normal[1]

[1] Based on the shapes of histograms.

Table 4-11
Summary of Reported Variabilities for Hardened Portland Cement Concrete

Property	Standard Deviation	Coefficient of Variation, Percent	Normality
Density in hardened state, kg/m^3	N/A	2	not reported[1]
Compressive strength, MPa	4	15	normal[2]
Compressive modulus, MPa	N/A	30	not reported[1]
Poisson's ratio	N/A	15	not reported[1]
Indirect tensile strength, kPa	570	15	normal[2]
Indirect tensile modulus, MPa	8720	35	normal[2]
Flexural strength, kPa	350	7	normal[2]
Slab thickness, mm	9	3	normal[2]
Load transfer, percent			conflicts[3]
- doweled expansion	7	25	
- doweled construction	5	25	
- keyed construction	10	40	
- doweled contraction	4	4-55	
- tied contraction	5	20	
- plain contraction	4	25	
Falling weight deflectometer (FWD) Deflections - under load, μm	3-120	15	normal[2]
Falling weight deflectometer (FWD) Modulus calculations			not reported[1]
- PCC modulus, MPa	2830-15740	30	
- subgrade modulus, MPa	15-30	15	
Serviceability index	0.15	3	normal[2]

[1] Reports on variability did not address normality.
[2] Based on the shapes of histograms.
[3] Reports concerning normality conflicted.
N/A Limited data or not reported.

5 Summary, Conclusions, and Recommendations

Summary and Conclusions

The paving industry is moving forward with performance-related specifications that incorporate statistical concepts for the development of acceptance plans, including quality control and quality assurance testing. Personnel involved in pavement construction need to understand these statistical concepts and they need to know the levels of variability that should be expected in their test results.

This report provides information on statistical methods that can be used by pavement engineers and contractors to analyze the variability of construction materials. In addition, it summarizes selected published data on these variabilities. These data were measured within "lots" of material, where a lot is defined as a quantity of material that can be considered uniform for sampling and payment purposes.

The methods for analyzing variability will be useful for those who need to quantify the relative magnitudes of different sources of variation. If a contractor's product is highly variable, these methods can be used to determine if the production of materials is out of control or if the methods for sampling and/or testing are inconsistent. By keeping track of the trends in test results, the contractor will be able to adjust production and/or construction methods as necessary in order to provide a high-quality, consistent product.

The data pertaining to published variabilities provide the pavement designer and specification writer with tools for determining what can be reasonably expected of the contractor. A contractor cannot be expected to keep material variation well below the published norm. Some sources of variability, such as those inherent to a particular test method, are out of the contractor's control. Knowledge of expected variability will also help the owner control two important aspects of specifications: the risks for contractual parties and the reliability of the final product.

Recommendations

The purpose of this report was to review statistical concepts and compile data related to the variability of materials used for the construction of pavements. Based on the concepts presented and data reviewed, the following actions are recommended.

a. The current Corps of Engineers specification requirements for pavement construction should be reviewed to determine the appropriateness of the statistical concepts incorporated.

b. Where necessary, the Corps of Engineers specifications should be adjusted to reflect reasonable expectations of the contractor so that the risks assumed by the contractor and owner are equitable.

c. Corps of Engineer specifications that do not contain statistical concepts for acceptance testing should be modified.

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Appendix A
Statistical Reference Tables

Table A1
Probability of Obtaining a Random Value of Z Greater Than the Values Shown in the Margins (after Steel and Torrie 1980)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0078	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019

(Continued)

Table A1 (Concluded)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001

Table A2
Values of *t* (after Fisher and Yates 1949)

df	Probability of a Numerically Larger Value of *t* (Two-Tail Test)				
	0.1	**0.05**	**0.02**	**0.01**	**0.001**
1	6.314	12.706	31.821	63.657	636.619
2	2.920	4.303	6.965	9.925	31.598
3	2.353	3.182	4.541	5.841	12.941
4	2.132	2.776	3.747	4.604	8.610
5	2.015	2.571	3.365	4.032	6.859
6	1.943	2.447	3.143	3.707	5.959
7	1.895	2.365	2.998	3.499	5.405
8	1.860	2.306	2.896	3.355	5.041
9	1.833	2.262	2.821	3.250	4.781
10	1.812	2.228	2.764	3.169	4.587
11	1.796	2.201	2.718	3.106	4.437
12	1.782	2.179	2.681	3.055	4.318
13	1.771	2.160	2.650	3.012	4.221
14	1.761	2.145	2.624	2.977	4.140
15	1.753	2.131	2.602	2.947	4.073
16	1.746	2.120	2.583	2.921	4.015
17	1.740	2.110	2.567	2.898	3.965
18	1.734	2.101	2.552	2.878	3.922
19	1.729	2.093	2.539	2.861	3.883
20	1.725	2.086	2.528	2.845	3.850
df	**0.05**	**0.025**	**0.01**	**0.005**	**0.0005**
	Probability of a Larger Positive Value of *t* (One-Tail Test)				

(Continued)

Table A2 (Concluded)

df	Probability of a Numerically Larger Value of *t* (Two-Tail Test)				
	0.1	0.05	0.02	0.01	0.001
21	1.721	2.080	2.518	2.831	3.819
22	1.717	2.074	2.508	2.819	3.792
23	1.714	2.069	2.500	2.807	3.767
24	1.711	2.064	2.492	2.797	3.745
25	1.708	2.060	2.485	2.787	3.725
26	1.706	2.056	2.479	2.779	3.707
27	1.703	2.052	2.473	2.771	3.690
28	1.701	2.048	2.467	2.763	3.674
29	1.699	2.045	2.462	2.756	3.659
30	1.697	2.042	2.457	2.750	3.646
40	1.684	2.021	2.423	2.704	3.551
60	1.671	2.000	2.390	2.660	3.460
120	1.658	1.980	2.358	2.617	3.373
∞	1.645	1.960	2.326	2.576	3.291
df	0.05	0.025	0.01	0.005	0.0005
	Probability of a Larger Positive Value of *t* (One-Tail Test)				

Table A3
Values of χ^2 (after Thompson 1941)

df	Probability of Obtaining a Larger Value of χ^2										
	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010
1	< .0001	< .0001	.0004	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63
2	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21
3	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3
4	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3
5	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1
6	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8
7	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5
8	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1
9	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7
10	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2
11	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7
12	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2
13	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7
14	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1
15	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6
16	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0
17	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4

(Continued)

Table A3 (Concluded)

df	Probability of Obtaining a Larger Value of χ^2										
	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010
18	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8
19	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2
20	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6
21	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9
22	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3
23	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6
24	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0
25	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3
26	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6
27	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0
28	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3
29	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6
30	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9
40	22.2	24.4	26.5	29.1	33.7	39.3	45.6	51.8	55.8	59.3	63.7
50	29.7	32.4	34.8	37.7	42.9	49.3	56.3	63.2	67.5	71.4	76.2
60	37.5	40.5	43.2	46.5	52.3	59.3	67.0	74.4	79.1	83.3	88.4

Table A4
Values of *F* (after Steel and Torrie 1980)

Denominator *df*	Problem of a Larger *F*	Numerator *df* 1	2	4	6
1	.050	161.4	199.5	224.6	234.0
	.025	647.8	799.5	899.6	937.1
	.010	4,052	5,000	5,625	5,859
2	.050	18.51	19.00	19.25	19.33
	.025	38.51	39.00	39.25	39.33
	.010	98.50	99.00	99.25	99.33
4	.050	7.71	6.94	6.39	6.16
	.025	12.22	10.65	9.60	9.20
	.010	21.20	18.00	15.98	15.21
6	.050	5.99	5.14	4.53	4.28
	.025	8.81	7.26	6.23	5.82
	.010	13.75	10.92	9.15	8.47
8	.050	5.32	4.46	3.84	3.58
	.025	7.57	6.06	5.05	4.65
	.010	11.26	8.65	7.01	6.37
10	.050	4.96	4.10	3.48	3.22
	.025	6.94	5.46	4.47	4.07
	.010	10.04	7.56	5.99	5.39
20	.050	4.35	3.49	2.87	2.60
	.025	5.87	4.46	3.51	3.13
	.010	8.10	5.85	4.43	3.87
∞	.050	3.84	3.00	2.37	2.10
	.025	5.02	3.69	2.79	2.41
	.010	6.63	4.61	3.32	2.80

(Continued)

Table A4 (Concluded)

Denominator *df*	Prob. of a Larger *F*	Numerator *df* 8	10	20	∞
1	.050	238.9	241.9	248.0	254.3
	.025	956.7	968.6	993.1	1018
	.010	5,982	6,056	6,209	6,366
2	.050	19.37	19.40	19.45	19.50
	.025	39.37	39.40	39.45	39.50
	.010	99.37	99.40	99.45	99.50
4	.050	6.04	5.96	5.80	5.63
	.025	8.98	8.84	8.56	8.26
	.010	14.80	14.55	14.02	13.46
6	.050	4.15	4.06	3.87	3.67
	.025	5.60	5.46	5.17	4.85
	.010	8.10	7.87	7.40	6.88
8	.050	3.44	3.35	3.15	2.93
	.025	4.43	4.30	4.00	3.67
	.010	6.03	5.81	5.36	4.86
10	.050	3.07	2.98	2.77	2.54
	.025	3.85	3.72	3.42	3.08
	.010	5.06	4.85	4.41	3.91
20	.050	2.45	2.35	2.12	1.84
	.025	2.91	2.77	2.46	2.09
	.010	3.56	3.37	2.94	2.42
∞	.050	1.94	1.83	1.57	1.00
	.025	2.19	2.05	1.71	1.00
	.010	2.51	2.32	1.88	1.00

Table A5
Critical Values for T When Standard Deviation is Calculated from the Sample [after ASTM (1995b) E 178]

No. of Observed	Significance Level (One-Sided Test)					
	0.1%	0.5%	1.0%	2.5%	5%	10%
3	1.155	1.155	1.155	1.155	1.153	1.148
4	1.499	1.496	1.492	1.481	1.463	1.425
5	1.780	1.764	1.749	1.715	1.672	1.602
6	2.011	1.973	1.944	1.887	1.822	1.729
7	2.201	2.139	2.097	2.020	1.938	1.828
8	2.358	2.274	2.221	2.126	2.032	1.909
9	2.492	2.387	2.323	2.215	2.110	1.977
10	2.606	2.482	2.410	2.290	2.176	2.036
15	2.997	2.806	2.705	2.549	2.409	2.247
20	3.230	3.001	2.884	2.709	2.557	2.385
25	3.389	3.135	3.009	2.822	2.663	2.486
30	3.507	3.236	3.103	2.908	2.745	2.563
40	3.673	3.381	3.240	3.036	2.866	2.682
50	3.789	3.483	3.336	3.128	2.956	2.768
60	3.874	3.560	3.411	3.199	3.025	2.837
70	3.942	3.622	3.471	3.257	3.082	2.893
80	3.998	3.673	3.521	3.305	3.130	2.940
90	3.998	3.673	3.521	3.305	3.130	2.940
100	4.084	3.754	3.600	3.383	3.207	3.017
110	4.044	3.716	3.563	3.347	3.171	2.981
120	4.150	3.817	3.662	3.444	3.267	3.078
130	4.178	3.843	3.688	3.470	3.294	3.104
140	4.203	3.867	3.712	3.493	3.318	3.129

Table A6
Tietjen-Moore Critical Values (x1000) for E_k [after ASTM (1995b) E 178]

k	α	Number of Values in Data Set												
		50	40	30	20	15	10	9	8	7	6	5	4	3
1	0.01	748	704	624	499	404	235	197	156	110	68	29	4	N/A
	0.05	796	756	698	594	503	353	310	262	207	145	81	25	1
	0.10	820	784	730	638	556	415	374	326	270	203	127	49	3
2	0.01	636	574	482	339	238	101	78	50	28	12	2	N/A	N/A
	0.05	684	629	549	416	317	172	137	99	65	34	10	1	N/A
	0.10	708	657	582	460	360	214	175	137	94	56	22	2	N/A
3	0.01	550	480	386	236	146	44	26	14	6	1	N/A	N/A	N/A
	0.05	599	534	443	302	206	83	57	34	16	4	N/A	N/A	N/A
	0.10	622	562	475	338	240	108	80	53	27	9	N/A	N/A	N/A
4	0.01	482	408	308	170	90	18	9	4	N/A	N/A	N/A	N/A	N/A
	0.05	529	458	364	221	134	37	21	10	N/A	N/A	N/A	N/A	N/A
	0.10	552	486	391	252	160	52	32	16	N/A	N/A	N/A	N/A	N/A
5	0.01	424	347	250	121	54	6	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	0.05	468	395	298	163	84	14	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	0.10	492	422	325	188	105	22	N/A	N/A	N/A	N/A	N/A	N/A	N/A

k = Number of potential outliers.
α = Level of significance.
N/A Not applicable.

Table A7
Values of F_{max} (after Ott 1977)

df each sample	Problem of a Larger F_{max}	Number of Independent Mean Squares				
		2	3	4	5	6
2	.050	39.0	87.5	142	202	266
	.010	199	448	729	1,036	1,362
3	.050	15.4	27.8	39.2	50.7	62.0
	.010	47.5	85	120	151	184
4	.050	9.60	15.5	20.6	26.2	29.5
	.010	23.2	37	49	59	69
5	.050	7.15	10.3	13.7	16.3	18.7
	.010	14.9	22	28	33	38
6	.050	5.82	8.38	10.4	12.1	13.7
	.010	11.1	15.5	19.1	22	25
7	.050	4.99	6.94	8.44	9.70	10.8
	.010	8.89	12.1	14.5	16.5	18.4
8	.050	4.43	6.00	7.18	8.12	9.03
	.010	7.50	9.9	11.7	13.2	14.5
9	.050	4.03	5.34	6.31	7.11	7.80
	.010	6.54	8.5	9.9	11.1	12.1
10	.050	3.72	4.85	5.67	6.34	6.92
	.010	5.85	7.4	8.6	9.6	10.4
12	.050	3.28	4.16	4.79	5.30	5.72
	.010	4.91	6.1	6.9	7.6	8.2
15	.050	2.86	3.54	4.01	4.37	4.68
	.010	4.07	4.9	5.5	6.0	6.4
30	.050	2.07	2.40	2.61	2.78	2.91
	.010	2.63	3.0	3.3	3.4	3.6
∞	.050	1.00	1.00	1.00	1.00	1.00
	.010	1.00	1.00	1.00	1.00	1.00

(Continued)

Table A7 (Concluded)

df each sample	Problem of a Larger F_{max}	Number of Independent Mean Squares				
		7	8	10	12	15
2	.050	333	403	550	704	968
	.010	1,705	2,063	2,813	3,605	4,873
3	.050	72.9	83.5	104	124	151
	.010	216	249	310	361	418
4	.050	33.6	37.5	44.6	51.4	58.1
	.010	79	89	106	120	137
5	.050	20.8	22.9	26.5	29.9	32.6
	.010	42	46	54	60	66
6	.050	15.0	16.3	18.6	20.7	22.1
	.010	27	30	34	37	39
7	.050	11.8	12.7	14.3	15.8	16.6
	.010	20	22	24	27	32
8	.050	9.8	10.5	11.7	12.7	13.1
	.010	15.8	16.9	18.9	21	24
9	.050	8.41	8.95	9.91	10.7	10.9
	.010	13.1	13.9	15.3	16.6	18.2
10	.050	7.42	7.87	8.66	9.34	9.45
	.010	11.1	11.8	12.9	13.9	15.1
12	.050	6.09	6.42	7.00	7.48	8.03
	.010	8.7	9.1	9.9	10.6	11.6
15	.050	4.95	5.19	5.59	5.93	6.31
	.010	6.7	7.1	7.5	8.0	8.8
30	.050	3.02	3.12	3.29	3.39	3.40
	.010	3.7	3.8	4.0	4.2	4.5
∞	.050	1.00	1.00	1.00	1.00	1.00
	.010	1.00	1.00	1.00	1.00	1.00

Appendix B
Residual Fine-Grained Soil Deposits

Table B1
Specific Gravity

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
2.86	0.03	1.0	Schultze (1972)[1]
N/A	N/A	2.0	Padilla and Vanmarcke (1974)

[1] Normality was not rejected by the chi square "goodness-of-fit" test.
N/A Data not reported.

Table B2
Natural Water Content, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
14	3.5	26	Krahn and Fredlund (1983)
21	4.7	23	Schultze (1972)[1]
22	2.8	13	
27	6.0	22	Fredlund and Dahlman (1972)[2]
29	5.8	20	Krahn and Fredlund (1983)
29	5.1	18	Fredlund and Dahlman (1972)[2]
32	4.4	14	Krahn and Fredlund (1983)
33	4.1	12	Fredlund and Dahlman (1972)[2]
35	3.5	10	
35	4.6	13	
36	4.3	12	

[1] Normality was rejected by the chi square "goodness-of-fit" test.
[2] Frequency distributions appeared to be normal in shape.

Table B3
Porosity

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
0.33	0.06	19	(Schultze 1972)[1]
0.38	0.05	13	
0.39	0.04	10	
0.40	0.03	7.5	
0.48	0.05	10	

[1] Normality was not rejected by the chi square "goodness-of-fit" test.

Table B4
Void Ratio

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
0.42	0.09	21	(Krahn and Fredlund 1983)
0.51	0.09	18	(Schultze 1972)[1]
0.53	0.14	26	
0.56	0.17	30	
0.57	0.10	17	
0.66	0.09	14	
0.75	0.24	32	(Fredlund and Dahlman 1972)
0.90	0.16	18	
0.91	0.14	15	
0.91	0.19	20	
0.95	0.13	14	(Krahn and Fredlund 1983)

[1] Normality was not rejected by the chi square "goodness-of-fit" test.

Table B5
Degree of Saturation, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
78.7	11.4	15	Krahn and Fredlund (1983)
81.3	10.6	13	
85	16	19	Schultze (1972)[1]
91.5	6.0	6.6	Krahn and Fredlund (1983)
91.9	12.1	13	Fredlund and Dahlman (1972)
93.3	8.0	8.5	
94.1	10.0	11	Krahn and Fredlund (1983)
95.6	8.6	9.0	Fredlund and Dahlman (1972)
97.5	8.4	8.6	

[1] Normality was rejected by the chi square "goodness-of-fit" test.

Table B6
Dry Density, kg/m^3

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
1,430	99	6.9	(Krahn and Fredlund 1983)
1,880	106	5.6	

Table B7
Coefficient of Permeability, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Comment	Reference
N/A	N/A	90	80 percent saturated	Nielson et al. (1973)
N/A	N/A	700	100 percent saturated	

N/A Data not reported.

Table B8
Liquid Limit, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
16	9.7	62	Mitchell et al. (1977)
20	4.0	20	Kennedy, Hudson, and McCullough (1975)
27	5.4	20	
27	6.0	22	Krahn and Fredlund (1983)
33	6.3	19	Kennedy, Hudson, and McCullough (1975)
43	11	26	Wahls and Futrell (1966)
53	11	21	Fredlund and Dahlman (1972)[1]
55	12	22	Schultze (1972)[2]
57	13	24	Krahn and Fredlund (1983)
59	13	22	Fredlund and Dahlman (1972)[1]
62	12	19	Krahn and Fredlund (1983)
63	14	22	Wahls and Futrell (1966)
63	11	18	Fredlund and Dahlman (1972)[1]
64	11	17	
73	13	18	Krahn and Fredlund (1983)

[1] Frequency distributions appeared normal in shape.
[2] Normality was not rejected by the chi square "goodness-of-fit" test.
Note: Sampling error and experimental error (testing and inherent) accounted for 40 percent of the total variance (Hampton, Yoder, and Burr 1962).

Table B9
Plastic Limit, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
14	2.2	16	Krahn and Fredlund (1983)
21	3.4	16	Schultze (1972)[1]
23	3.8	17	Krahn and Fredlund (1983)
24	3.2	13	Fredlund and Dahlman (1972)[2]
25	3.8	15	
25	3.2	13	
25	3.4	13	Krahn and Fredlund (1983)
26	3.4	13	Fredlund and Dahlman (1972)[2]
27	4.8	19	Krahn and Fredlund (1983)

[1] Normality was rejected by the chi square "goodness-of-fit" test.
[2] Frequency distributions appeared normal in shape.
Note: Sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance (Hampton, Yoder, and Burr 1962).

Table B10
Plasticity Index, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.3	3.4	80	Mitchell et al. (1977)
10	4.8	48	Kennedy, Hudson, and McCullough (1975)
12	7.2	59	Wahls and Futrell (1966)
14	5.9	42	Kennedy, Hudson, and McCullough (1975)
16	2.6	16	
27	10.2	39	Wahls and Futrell (1966)
33.9	9.6	28	Schultze (1972)[1]

[1] Normality was rejected by the chi square "goodness-of-fit" test.
Notes: (1) Frequency distributions appeared normal in shape (Ingles 1972).
(2) Sampling error and experimental error (testing and inherent) accounted for 60 percent of the total variance (Hampton, Yoder, and Burr 1962).

Table B11
Particle Size Distribution, Percent Finer

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
No. 40	79	16	21	Wahls and Futrell (1966)
	91	9.4	10	
No. 200	14	2.4	17	Auff and Choummanivong (1994)
	56	20	36	Wahls and Futrell (1966)
	77	7.0	9.1	Hampton, Yoder, and Burr (1962)[1]
	79	16	20	Wahls and Futrell (1966)
2 μm	17	11	62	Wahls and Futrell (1966)
	27	3.5	13	Hampton, Yoder, and Burr (1962)[2]
	41	12	30	Wahls and Futrell (1966)

[1] Sampling error and experimental error (testing and inherent) accounted for 50 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 65 percent of the total variance.

Table B12
Potential for Volume Change

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Compression index	0.108	0.032	30	Krahn and Fredlund (1983)
	0.110	0.052	47	Fredlund and Dahlman (1972)
	0.155	0.049	33	Krahn and Fredlund (1983)
	0.159	0.048	30	Fredlund and Dahlman (1972)
	0.167	0.048	29	
	0.184	0.047	26	
	0.205	0.066	32	Krahn and Fredlund (1983)
	0.265	0.070	26	
Swell index	0.032	0.017	53	Fredlund and Dahlman (1972)
	0.065	0.034	52	
	0.065	0.020	31	
	0.065	0.017	26	

Table B13
Moisture-Density Relationship

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Optimum moisture content, percent	19.8	3.5	18	Willenbrock (1974)[1]
	20.3	5.6	28	Wahls and Futrell (1966)
	26.4	5.4	20	
Maximum dry density, kg/m^3	1,495	132	8.8	Wahls and Futrell (1966)
	1,655	164	9.9	
	1,690	98	5.8	Willenbrock (1974)[2]

[1] Frequency distributions appeared normal in shape.
[2] Frequency distributions appeared to have negative skew.
Note: Sampling error and experimental error (testing and inherent) accounted for 40 percent and 25 percent of the total variance for optimum moisture content and maximum density, respectively (Hampton, Yoder, and Burr 1962).

Table B14
Unconfined Compressive Strength (kPa)

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
21	7.0	33	Ladd, Moe, and Gifford (1972)
26	14	53	Schultze (1972)[1]
40	21	52	
58	23	39	
125	59	48	Fredlund and Dahlman (1972)[2]
143	57	40	
161	66	41	
167	75	45	Krahn and Fredlund (1983)
199	98	49	Fredlund and Dahlman (1972)[2]
220	122	55	Krahn and Fredlund (1983)
253	179	71	
331	86	26	Morse (1972)
370	188	51	Krahn and Fredlund (1983)

[1] Normality was rejected by the chi square "goodness-of-fit" test.
[2] Frequency distribution appeared to have a positive skew.
Note: Frequency distributions appeared normal in shape (Ingles 1972).

Table B15
Measures of Shear Strength

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
California bearing ratio	10 percent	2.7 percent	27	Hampton, Yoder, and Burr (1962)[1]
Field vane	26 kPa	7.6 kPa	29 (19 to 35)	Ladd, Moe, and Gifford (1972)
Undrained triaxial test, c	N/A	N/A	17 to 43[2]	Kay and Krizek (1972)
	N/A	N/A	19	Lumb (1972)
Undrained triaxial test, $\tan\phi$	N/A	N/A	22	
Drained triaxial test, c	N/A	N/A	14	
Drained triaxial test, $\tan\phi$	N/A	N/A	1.6	
Drained shear box test, c	N/A	N/A	95 to 103[3,4]	
Drained shear box test, $\tan\phi$	N/A	N/A	18 to 46[3,5]	

[1] Sampling error and experimental error (testing and inherent) accounted for 80 percent of the total variance.
[2] Depends on sampling technique.
[3] Depends on type of soil.
[4] Sampling error and experimental error (testing and inherent) accounted for 20 to 50 percent of the total variance.
[5] Sampling error and experimental error (testing and inherent) accounted for 10 to 25 percent of the total variance.
N/A Data not reported.

Appendix C
Engineered Fill

Table C1
Controlled Moisture Content, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
8.3	2.6	31	Auff and Choummanivong (1994)
9.1	1.3	14	Selig (1966)
9.2	1.0	10	Yeo and Auff (1995)
11	1.1	11	
12	2.7	23	Auff and Choummanivong (1994)
12	2.0	16	Selig (1966)
15	3.6	24	David (1967)[1]

[1] Frequency distribution appeared to have positive skew.
Note: Frequency distributions appeared normal in shape (Baecher 1987).

Table C2
Field Compaction, Percent Relative to Laboratory Density

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
87.0	2.5	2.8	Yeo and Auff (1995)
87.2	6.1	7.0	Jorgensen (1968)[1]
88.7	4.5	5.1	
88.9	4.1	4.6	
89.9	8.8	9.8	
90.5	3.1	3.4	Sherman, Watkins, and Prysock (1966)
91.5	3.8	4.2	Yeo and Auff (1995)
92.8	2.9	3.1	Auff and Choummanivong (1994)
92.8	4.8	5.2	
92.9	2.4	2.6	Sherman, Watkins, and Prysock (1966)[2]
93.6	5.5	5.9	
94.5	3.0	3.2	Brown (1975)
96.6	2.9	2.9	Mitchell et al. (1977)
96.8	2.5	2.6	
96.8	5.7	5.9	Williamson and Yoder (1967)[3]
97.3	3.7	3.8	David (1967)[3]
97.8	4.9	5.0	Jorgensen (1968)
98.2	4.5	4.6	Williamson and Yoder (1967)
98.2	3.2	3.3	Mitchell et al. (1977)
99.0	4.6	4.6	Nielson (1967)[4]
99.1	4.5	4.5	Van Houten (1967)[1]
100.6	5.3	5.3	Williamson and Yoder (1967)[2]

[1] Sampling error and experimental error (testing and inherent) accounted for an average of 50 percent of the total variance when tested by water balloon.
[2] Frequency distributions did not appear to be normal in shape.
[3] Frequency distributions appeared to be normal in shape.
[4] Sampling error and experimental error (testing and inherent) accounted for an average of 10 percent of the total variance when tested by sand cone.

Table C3
Resilient Modulus, MPa

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
N/A	N/A	20	Ingles (1972)
83	24	29	Kennedy, Hudson,and McCullough (1975)
110	24	22	
131	29	22	

N/A Data not reported.

Table C4
Measures of Stiffness and Strength

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
California bearing ratio, percent	7	1.5	22	Yoder and Witczak (1975)
	8	1.4	18	
	12	4.1	34	Yeo and Auff (1995)[1]
	18	4.7	26	Yoder and Witczak (1975)
	21	6.7	32	Yeo and Auff (1995)
	43	15	35	Mitchell et al. (1977)
Modulus of subgrade reaction, MPa/mm	N/A	N/A	33	Yoder and Witczak (1975)
	N/A	N/A	35	Highway Research Board (1962b)
	N/A	N/A	41	Kennedy, Hudson, and McCullough (1975)
Benkelman beam deflections, mm	1.4	0.35	25	Yeo and Auff (1995)
Dynaflect tests, MPa	N/A	N/A	34	Kennedy, Hudson, and McCullough (1975)

[1] Estimated by dynamic cone penetrometer (DCP) tests.
N/A Data not reported.

Table C5
Measures of Strength for Lime-Soil Mixtures and Soil-Cement

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Unconfined compressive strength, MPa	N/A	N/A	11 (1.3 to 30)	Liu and Thompson (1966)
Split-tensile strength, MPa	N/A	N/A	12 (5.0 to 23)	
Flexural strength, MPa	N/A	N/A	11 (3.2 to 19)	

N/A Data not reported.

Appendix D
Subbase and Base Course Materials

Table D1
Controlled Moisture Content, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.2	0.25	6.0	Auff and Yeo (1992)
4.5	0.26	5.8	
5.0	0.36	7.2	
5.0	0.63	12.6	Auff and Laksmanto (1994)
5.1	0.55	10.8	
5.1	0.55	10.8	Auff and Yeo (1992)
5.8	0.61	10.5	Auff and Laksmanto (1994)
5.9	0.75	12.7	Auff and Laksmanto (1993)
6.6	0.58	8.8	Auff and Laksmanto (1994)
6.6	0.40	6.1	Auff and Choummanivong (1994)
6.8	0.61	8.9	Highway Research Board (1962a)
7.0	1.1	15.7	Auff and Choummanivong (1994)
7.3	1.2	16.3	Auff and Laksmanto (1993)
7.4	0.40	5.4	
7.9	0.73	9.2	Yeo and Auff (1995)
8.2	2.0	24.4	Auff and Choummanivong (1994)
8.3	0.49	5.9	Auff and Laksmanto (1993)
8.7	0.38	4.4	Yeo and Auff (1995)
9.4	0.62	6.6	
9.6	0.74	7.7	
9.8	1.0	10.2	Auff and Choummanivong (1994)

Table D2
Atterberg Limits, Percent

Property	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Liquid limit	12	9.8	81	Mitchell et al. (1977)
	16	20	128	
	17	2.2	13	
	18	6.4	37	
	20	1.8	9.3	
Plasticity index	2.9	1.0	33	Mitchell et al. (1977)
	3.0	1.3	44	
	3.5	2.3	65	
	3.9	2.6	67	
	4.0	1.0	26	Ingles (1972)[1]
	5.9	1.4	24	Mitchell et al. (1977)

[1] Frequency distribution appeared to have positive skew.

Table D3
Coefficient of Permeability, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Comment	Reference
N/A	N/A	240	Silty sand	Lumb (1972)[1]

[1] Sampling error and experimental error (testing and inherent) accounted for 1 percent of the total variance when tested by constant head permeameter.
N/A Data not reported.

Table D4
Particle Size Distribution, Percent Finer

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
19-mm	20	0.9	4.5	Yeo and Auff (1995)
	22	2.4	11	
	22	2.2	10	
	22	2.5	11	
	70	7.3	10	Willenbrock (1974b)
	80	3.9	4.9	Kelley (1969)[1]
	90	2.5	2.8	
13.2-mm	85	2.3	2.7	Auff and Yeo (1992)
	86	1.8	2.1	
9.5-mm	42	8.1	19	Willenbrock (1974b)
	50	5.2	10	Kelley (1969)[2]
	70	2.9	4.1	Auff and Yeo (1992)
	71	2.2	3.1	
No. 4 Sieve	30	6.8	22	Willenbrock (1974b)
	34	4.3	13	Kelley (1969)[1]
	45	6.6	15	State of California (1967)
	50	4.3	8.7	
	51	3.1	6.1	
	53	5.7	11	
	53	2.9	5.5	Auff and Yeo (1992)
	54	4.1	7.6	Sherman (1971)[2]
	55	4.9	8.9	Kelley (1969)
	55	2.0	3.6	Auff and Yeo (1992)
	56	5.8	10	Sherman (1971)[3]
	58	2.8	4.8	State of California (1967)
	73	6.5	9.0	

(Sheet 1 of 3)

[1] Sampling error and experimental error (testing and inherent) accounted for 40 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance.
[3] Sampling error and experimental error (testing and inherent) accounted for 20 percent of the total variance.

Table D4 (Continued)				
Sieve Size	**Mean**	**Standard Deviation**	**Coefficient of Variation, Percent**	**Reference**
No. 8 Sieve[4]	27	3.9	15	Mitchell et al. (1977)
	32	4.0	12	
	38	2.3	6.1	Auff and Yeo (1992)
	39	2.0	5.1	
No. 16	20	3.6	18	Kelley (1969)[1]
	21	5.2	25	Willenbrock (1974b)
No. 30	24	2.9	12	State of California (1967)
	24	2.5	11	
	25	2.6	10	Sherman (1971)[2]
	27	2.3	8.4	State of California (1967)
No. 50	11	2.8	25	Kelley (1969)[3]
	14	2.4	17	Mitchell et al. (1977)
	16	2.5	15	
	18	1.0	5.6	Auff and Yeo (1992)
	18	1.7	9.4	
	37	6.3	17	Kelley (1969)
No. 100	5.5	1.7	31	Willenbrock (1974b)
	6	2.7	45	Kelley (1969)[4]
No. 200	4.6	1.0	22	State of California (1967)
	4.8	1.0	21	Auff and Choummanivong (1994)
	5.7	1.2	21	Mitchell et al. (1977)
	6.0	0.7	12	State of California (1967)
	6.2	0.9	15	Sherman (1971)[1]

(Sheet 2 of 3)

[1] Sampling error and experimental error (testing and inherent) accounted for 40 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance.
[3] Sampling error and experimental error (testing and inherent) accounted for 20 percent of the total variance.
[4] Sampling error and experimental error (testing and inherent) account for 30 percent of the total variance (Liu and Thompson 1966).

Table D4 (Concluded)

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
No. 200 (Cont.)	7.4	1.9	26	Auff and Choummanivong (1994)
	7.8	1.3	17	State of California (1967)
	7.9	1.1	14	
	8.6	1.7	20	
	8.8	1.6	18	Sherman (1971)[2]
	9.0	0.6	6.7	Auff and Yeo (1992)
	9.0	0.8	9.0	
	9.0	0.6	6.7	Auff and Laksmanto (1994)
	10	1.1	11	
	10	1.8	18	State of California (1967)
	13	1.2	9.2	Auff and Laksmanto (1994)
	13	0.9	7.0	Auff and Choummanivong (1994)
	16	1.8	11	
	16	2.9	18	Kelley (1969)
	20	0.9	4.5	Yeo and Auff (1995)
	21	2.4	11	
	22	2.2	10	
	22	2.5	11	

(Sheet 3 of 3)

[2] Sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance.

Table D5
Sand Equivalence, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
29	2.7	9.2	State of California (1967)
30	4.0	13	
31	6.1	20	
32	5.1	16	Sherman (1971)[1]
36	8.5	23	State of California (1967)
43	4.0	9.3	
44	4.7	11	Sherman (1971)[2]
59	4.0	6.8	State of California (1967)

[1] Sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 20 percent of the total variance.

Table D6
Magnesium Sulfate Soundness, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
5.6	0.48	8.5	New York State Department of Public Works (1964)
17	0.22	1.3	
23	0.24	1.0	
47	1.17	2.5	
48	1.46	3.1	

Table D7
Lift Thickness, mm

Mean	Standard Deviation	Coefficient of Variation, Percent	Comment	Reference
134	8.4	6.3	Subbase[1]	Mitchell et al. (1977)
307	35	11		Auff and Choummanivong (1994)
343	54	16		
93	17	18	Base[1]	Auff and Laksmanto (1994)
97	5.4	5.6		Auff and Yeo (1992)
98	13	14		Auff and Laksmanto (1994)
103	6.2	6.0		Auff and Yeo (1992)
104	23	22		Auff and Laksmanto (1994)
114	11	9.6		Mitchell et al. (1977)
131	11	8.3		
131	18	14		Auff and Choummanivong (1994)
133	19	14		
145	29	20		Auff and Laksmanto (1993)
147	27	18		

[1] Both unstabilized and stabilized.

Table D8
Field Compaction for Subbase Materials[1], Percent Relative to Laboratory Density

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
89.4	3.3	3.7	Williamson and Yoder (1967)
91.7	3.1	3.4	
93.6	2.3	2.5	
94.8	3.5	3.7	Brown (1975)
98.1	2.9	3.0	Nielson (1967)[2]
98.2	2.1	2.2	Mitchell et al. (1977)
98.7	2.9	3.0	Van Houten (1967)[3]
99.4	2.4	2.4	Mitchell et al. (1977)
99.4	2.1	2.1	Auff and Choummanivong (1994)
100.1	1.2	1.2	Auff and Laksmanto (1993)
100.8	2.3	2.3	David (1967)
100.9	3.8	3.8	Auff and Choummanivong (1994)
105.3	1.2	1.1	Auff and Laksmanto (1993)

[1] Both unstabilized and stabilized.
[2] Sampling error and experimental error (testing and inherent) accounted for 15 percent of the total variance.
[3] Sampling error and experimental error (testing and inherent) accounted for 25 percent of the total variance.

Table D9
Field Compaction for Base Course Materials[1], Percent Relative to Laboratory Density

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
94.3	2.2	2.3	Yeo and Auff (1995)
95.6	1.5	1.6	Brown (1975)
96.6	1.3	1.3	
97.5	2.1	2.2	Mitchell et al. (1977)
97.8	1.0	1.0	Brown (1975)
98.5	2.5	2.5	
99.2	4.1	4.1	David (1967)
99.2	0.7	0.7	Brown (1975)
99.5	2.8	2.8	Yeo and Auff (1995)
99.6	1.3	1.3	Mitchell et al. (1977)
100.0	2.0	2.0	
100.0	0.8	0.8	Brown (1975)
100.5	2.2	2.2	Yeo and Auff (1995)
100.7	1.9	1.9	Mitchell et al. (1977)
101.2	0.7	0.7	Auff and Yeo (1992)
101.3	2.2	2.2	Auff and Laksmanto (1994)
101.4	1.9	1.9	Yeo and Auff (1995)
101.6	2.8	2.8	Auff and Choummanivong (1994)
101.7	1.6	1.6	Auff and Laksmanto (1993)
102.3	0.7	0.7	Auff and Yeo (1992)
102.5	1.8	1.8	Auff and Laksmanto (1994)
103.0	1.4	1.4	Auff and Yeo (1992)
103.2	1.2	1.2	
103.3	1.9	1.8	Auff and Laksmanto (1993)
104.0	2.7	2.6	Auff and Choummanivong (1994)
104.2	2.2	2.1	Auff and Laksmanto (1994)
104.9	1.5	1.4	

[1] Both unstabilized and stabilized.

Table D10
Measures of Stiffness and Strength

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Angle of internal friction, ϕ	N/A	N/A	5 to 12	Schultze (1972)[1]
Unconfined compressive strength, kPa	N/A	N/A	11 (3 to 27)	Liu and Thompson (1966)
California bearing ratio, percent	26	8.3	32	Yoder and Witczak (1975)[2]
	59	13	22	Mitchell et al. (1977)[2]
	94	36	38	Yoder and Witczak (1975)[2]
	N/A	N/A	15 (7 to 26)	Liu and Thompson (1966)[3]
Benkelman beam deflections, mm	0.9	0.2	24	Yeo and Auff (1995)
	1.0	0.2	18	
	1.1	0.3	24	
	1.3	0.3	22	

[1] Normality was rejected by the chi square "goodness-of-fit" test, however, normality was not rejected for cotϕ .
[2] Performed in the field.
[3] Performed in the laboratory.
N/A Data not reported.

Table D11
Measures of Stiffness and Strength for Lime- and Cement-Stabilized Subbase and Base Course Materials

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Unconfined compressive strength, kPa	5,792	841	15	Highway Research Board (1962a)
	N/A	N/A	15 (4 to 24)	Lui and Thompson (1966)
Compression modulus, MPa	4,000	2,400	60	Marshall and Kennedy (1974)
Indirect tensile strength, kPa	938	338	36	Kennedy, Hudson, and McCullough (1975)[1]
Tensile modulus, MPa	621	421	68	
California bearing ratio, percent	216	81	38	Mitchell et al. (1977)[2]
	307	95	31	
	N/A	N/A	15 (3 to 31)	Lui and Thompson (1966)[3]
Plate-load tests, MPa/mm	N/A	N/A	71	Kennedy, Hudson, and McCullough (1975)
Dynaflect tests, MPa	N/A	N/A	22	

[1] Frequency distributions appeared to be normal in shape.
[2] Performed in the field.
[3] Performed in the laboratory.
N/A Data not reported.

Table D12
Measures of Stiffness and Strength for Asphalt-Stabilized Base Course Materials

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Indirect tensile strength, kPa	724	152	21	Marshall and Kennedy (1974)
Tensile modulus, MPa	401	144	36	
Indirect poisson's ratio	0.27	0.13	48	
Flexural stiffness, MPa	821	131	16[1]	Finn (1967)
	1,007	255	25[2]	
	1,014	283	28[2]	Monismith et al. (1967)
	3,034	607	20[2]	
	3,613	1,193	33[2]	Finn (1967)
	3,661	1,027	28[1]	
Fatigue life of beams	N/A	N/A	30 to 75[3]	Moore and Kennedy (1971)

[1] Laboratory-molded specimens.
[2] Specimens obtained from the field.
[3] Standard deviation increased linearly with increasing fatigue life.
Note: Frequency distributions for indirect tensile strength appeared to be normal in shape (Kennedy, Hudson, and McCullough 1975).
N/A Data not reported.

Appendix E
Asphalt Concrete

Table E1
Properties of Extracted Asphalt Cement

Property	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Penetration at 25°C, 0.1 mm	21	2.3	11	Sherman (1971)
	32	9.6	30	
	35	4.3	12	
	47	18	38	
	48	14	30	
Viscosity at 60°C, Pa-s	910	560	62	Sherman (1971)
	940	240	26	
	1,160	1,120	97	
	1,900	1,270	67	
	4,680	790	17	

Table E2
Mixture Temperature in the Field

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
135	7.4	5.5 (2.2 to 8.0)	Kilpatrick and McQuate (1967)
153	9.0	5.9	Oglio and Zenewitz (1965)[1]

[1] Sampling error and experimental error (testing and inherent) accounted for 2 percent of the total variance.

Table E3
Extracted Asphalt Cement Content, Percent

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.0	0.18	4.5	Shook (1966)
4.2	0.13	3.1	Highway Research Board. (1962a)
4.3	0.39	9.0	Shook (1966)
4.4	0.20	4.5	
4.5	0.30	6.6	Gartner (1965)
4.9	0.33	6.7	Hode-Keyser and Wade (1963)
5.1	0.38	7.5	Adams and Shah (1965)
5.2	0.18	3.5	Highway Research Board. (1962a)
5.3	0.18	3.4	Adams and Shah (1965)
5.3	0.28	5.2	Shook (1966)
5.4	0.17	3.1	
5.4	0.13	2.4	Mitchell et al. (1977)
5.4	0.16	3.0	Adams and Shah (1965)
5.6	0.14	2.6	Shook (1966)
5.8	0.39	6.8	Gartner (1965)
6.0	0.12	2.0	Shook (1966)
6.0	0.20	3.4	
6.0	0.12	2.0	Oglio and Zenewitz (1965)
6.2	0.35	5.7	Shook (1966)
6.4	0.29	4.5	
6.4	0.22	3.5	Hode-Keyser and Wade (1963)

Note: On the average, sampling error and experimental error (testing and inherent) accounted for 50 percent of the total variance (Oglio and Zenewitz 1965; Granley 1969; Shook 1966).

Table E4
Particle Size Distribution for Extracted Aggregates, Percent Finer

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
25 mm	94	4.5	4.8	SCSHD (1966)
19 mm	93	1.4	1.5	Granley (1969)
13 mm	92	3.2	3.4	SCSHD (1966)
	99	0.9	0.9	Mitchell et al. (1977)
9.5 mm	98	1.7	1.8	Oglio and Zenewitz (1965)
	86	2.5	2.9	Granley (1969)

(Sheet 1 of 4)

Note: Sampling error and experimental error (testing and inherent) accounted for the following percentages of total variance.
- 25-mm sieve: 85 percent (SCSHD 1966).
- 19-mm sieve: 75 percent (Granley 1969).
- 13-mm sieve: 65 percent (SCSHD 1966).
- 9.5-mm sieve: 60 percent (Granley 1969).

Table E4 (Continued)

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.75 mm	36	2.2	6.1	Highway Research Board (1962a)
	41	4.5	11	Shook (1966)
	43	4.5	11	SCSHD (1966)
	50	3.0	5.9	Hode-Keyser and Wade (1963)
	51	3.3	6.5	Shook (1966)
	54	3.5	6.4	Mitchell et al. (1977)
	60	3.4	5.7	Shook (1966)
	61	2.0	3.2	
	62	3.7	6.0	
	62	4.2	6.7	Oglio and Zenewitz (1965)
	62	4.1	6.7	SRCWV (1966)
	63	4.0	6.4	Highway Research Board (1962a)
	64	3.5	5.5	Granley (1969)
	65	4.3	6.6	SCSHD (1966)
	65	3.5	5.4	Adams and Shah (1965)
	65	1.6	2.4	Shook (1966)
	66	1.8	2.7	
	66	2.6	4.0	Adams and Shah (1965)
	66	4.7	7.2	SRCWV (1966)
	67	3.9	5.9	Shook (1966)
	68	4.8	7.1	SRCWV (1966)
	68	3.9	5.7	Adams and Shah (1965)
	69	4.7	6.8	
	69	4.5	6.5	SRCWV (1966)
	70	4.8	6.9	
	72	5.1	7.1	
	73	2.3	3.2	Shook (1966)
	78	2.7	3.5	
	88	0.8	0.9	Hode-Keyser and Wade (1963)

(Sheet 2 of 4)

Note: On the average, sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance (Granley 1969; SCSHD 1966; Oglio and Zenewitz 1965; Shook 1966; SRCWV 1966).

Table E4 (Continued)

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
2.36 mm	36	2.9	8.0	Mitchell et al. (1977)
	42	2.8	6.7	Granley (1969)
2.0 mm	35	3.7	11	SCSHD (1966)
	39	2.1	5.5	Oglio and Zenewitz (1965)
	55	4.0	7.4	SCSHD (1966)
0.85 mm	26	2.1	7.8	Oglio and Zenewitz (1965)
0.6 mm	26	1.7	6.7	Granley (1969)
0.425 mm	18	1.7	9.2	Oglio and Zenewitz (1965)
	20	3.6	18	SCSHD (1966)
	29	2.1	7.2	
0.3 mm	15	1.6	11	Mitchell et al. (1977)
	15	1.4	9.1	Granley (1969)
0.18 mm	9.5	1.5	15	Oglio and Zenewitz (1965)
0.15 mm	9.0	1.0	11	Granley (1969)

(Sheet 3 of 4)

Note: Sampling error and experimental error (testing and inherent) accounted for the following percentages of total variance.
- 2.36-mm sieve: 25 percent (Granley 1969).
- 2.0-mm sieve: 50 percent (SCSHD 1966).
- 0.6-mm sieve: 30 percent (Granley 1969).
- 0.425-mm sieve: 50 percent (SCSHD 1966).
- 0.30-mm sieve: 35 percent (Granley 1969).
- 0.15-mm sieve: 30 percent (Granley 1969).
- 0.075-mm sieve: 50 percent (SCSHD 1966).

Table E4 (Concluded)

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
No. 200	2.6	0.5	19	Hode-Keyser and Wade (1963)
	4.1	0.6	13	Shook (1966)
	4.3	0.5	11	Highway Research Board (1962a)
	4.3	0.7	16	SCSHD (1966)
	4.9	1.0	20	Shook (1966)
	5.6	1.2	21	Oglio and Zenewitz (1965)
	5.6	1.2	21	Shook (1966)
	5.8	1.1	20	
	5.9	1.2	20	Highway Research Board (1962a)
	6.0	0.9	16	Granley (1969)
	6.0	1.0	17	Shook (1966)
	6.4	1.0	16	SCSHD (1966)
	7.0	1.1	16	Adams and Shah (1965)
	7.1	0.8	11	Mitchell et al. (1977)
	7.1	1.0	14	Adams and Shah (1965)
	7.3	1.0	13	Shook (1966)
	7.5	1.3	17	Adams and Shah (1965)
	7.9	0.7	9.3	Shook (1966)
	8.0	1.3	16	Adams and Shah (1965)
	9.6	0.9	9.0	Shook (1966)
	10	1.0	9.5	
	10	0.8	8.1	Hode-Keyser and Wade (1963)

(Sheet 4 of 4)

Note: On the average, sampling error and experimental error (testing and inherent) accounted for 50 percent of the total variance (Granley 1969; SCSHD 1966).

Table E5
Laboratory Density and Voids Analyses

Property	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Bulk specific gravity	2.353	0.038	1.6	Highway Research Board (1962a)
	2.388	0.026	1.1	
Theo. Max. specific gravity	2.460	0.011	0.4	Granley (1969)[1]
Voids total mix, percent	1.8	0.8	44	Sherman (1971)
	2.3	1.5	65	
	2.4	0.4	17	
	2.5	1.5	60	
	4.3	1.0	23	Granley (1969)[2]
	4.9	0.8	17	Mitchell et al. (1977)
	5.7	0.9	16	Sherman (1971)
	6.5	1.5	23	Highway Research Board (1962a)
	7.7	1.0	13	
Voids filled, percent	56.5	3.5	6.2	Highway Research Board (1962a)
	65.4	5.3	8.1	
	70.0	3.7	5.3	Sherman (1971)
	85.6	2.2	2.6	
	86.8	7.5	8.6	
	87.4	6.8	7.8	
	89.4	4.3	4.8	

[1] Sampling error and experimental error (testing and inherent) accounted for 30 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 20 percent of the total variance.

Table E6
Field Compaction, Percent Relative to Laboratory Density

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
93.1	1.6	1.7	Granley (1969)[1]
94.2	2.9	3.1	
95.3	1.2	1.3	Shook (1966)[2]
95.6	1.4	1.5	
96.0	1.4	1.5	
96.2	1.1	1.1	Mitchell et al. (1977)
96.9	1.0	1.0	Highway Research Board (1962a)
97.0	1.5	1.5	
97.6	0.6	0.6[3]	Kennedy, Hudson, and McCullough (1975)
N/A	N/A	3.7[4]	

[1] Sampling error and experimental error (testing and inherent) accounted for 35 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 40 percent of the total variance.
[3] Airfield.
[4] Highway.
N/A Data not reported.

Table E7
Measures of Stiffness and Strength

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Marshall stability, kN	9.9	1.3	13	Mitchell et al. (1977)
	10.3	1.3	12	Granley (1969)[1]
Marshall flow, mm	2.2	0.3	15	Granley (1969)[2]
	2.3	0.3	15	Mitchell et al. (1977)
Indirect tensile strength, kPa	531	85	16	Marshall and Kennedy (1974)
Indirect static modulus, MPa	289	84	29	Marshall and Kennedy (1974)
	397	258	65	Kennedy, Hudson, and McCullough (1975)
	654	477	73	
Indirect poisson's ratio	0.40	0.11	27	Marshall and Kennedy (1974)
	N/A	N/A	52 (38 to 73)	Kennedy, Hudson, and McCullough (1975)
Dynamic modulus at 4°C, MPa	N/A	N/A	13 (9.2 to 16)	The Asphalt Institute (1974)
Dynamic modulus at 21°C, MPa	N/A	N/A	16 (11 to 19)	The Asphalt Institute (1974)
	N/A	N/A	40 (24 to 62)	Kennedy, Hudson, and McCullough (1975)
Dynamic modulus at 38°C, MPa	N/A	N/A	22 (21 to 23)	The Asphalt Institute (1974)

[1] Sampling error and experimental error (testing and inherent) accounted for 60 percent of the total variance.
[2] Sampling error and experimental error (testing and inherent) accounted for 75 percent of the total variance.
N/A Data not reported.

Table E8
Flexural Properties of Asphalt Concrete

Property	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Flexural stiffness at 4°C, Mpa	3,972	517	13	Finn (1967)[1]
	4,627	1,020	22	Finn (1967)[2]
	4,909	979	20	Monismith et al. (1967)[2]
Flexural stiffness at 21°C, MPa	889	152	17	Finn (1967)[1]
	N/A	N/A	13	Monismith et al. (1967)[1]
	1,048	283	27	Monismith et al. (1967)[2]
	1,089	276	25	Finn (1967)[2]
Fatigue life at 21°C, cycles	N/A	N/A	65 (23 to 135)	Monismith et al. (1970)[3]

[1] Laboratory-compacted samples.
[2] Field samples.
[3] Controlled stress.
N/A Data not reported.

Table E9
Mat Thickness

Mean, mm	Standard Deviation	Coefficient of Variation, Percent	Reference
66	5.6	8.4	Attoh-Okine and Roddis (1994)
71	6.6	9.3	
79	12	15	Keyser and Waell (1968)[1]
112	5.6	5.0	Attoh-Okine and Roddis (1994)
N/A	6.6	N/A	Granley (1969)
N/A	10	N/A	Huculak (1968)
N/A	10	N/A	Sherman (1971)
N/A	N/A	2.7 to 5.9	Yoder and Witczak (1975)[2]
N/A	N/A	3.5 to 19.2	Yoder and Witczak (1975)[3]
N/A	N/A	22	Yoder and Witczak (1975)[4]

[1] Frequency distributions appeared to be normal in shape.
[2] New airfield.
[3] New highway.
[4] Overlay.
N/A Data not reported.

Table E10
Benkelman Beam Deflections, mm

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
2.6	1.8	69	Kerr and Parkes (1966)[1]
3.2	1.3	41	
3.6	1.7	47	
N/A	N/A	15	C.G.R.A. (1962)[2]
N/A	N/A	22	C.G.R.A. (1962)[3]

[1] Frequency distributions appeared to have a positive skew.
[2] WASHO Road Test.
[3] State highways.
N/A Data not reported.

Table E11
Falling Weight Deflectometer Results

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Deflections under load, μm	182	30	16	Siddharthan, Sebaaly, and Javaregowda (1992)[1]
	212	28	13	
	728	124	17	Grogan (1991)
	1,082	341	32	
	1,293	153	12	Siddharthan, Sebaaly, and Javaregowda (1992)[1]
	1,303	158	12	
	1,307	81	6.2	Grogan (1991)
	1,473	287	20	
Deflections at 0.9 m offset, μm	55	7.9	14	Siddharthan, Sebaaly, and Javaregowda (1992)[1]
	60	11	18	
	173	29	17	
	174	26	15	
Backcalculated moduli for asphalt concrete, MPa	227	211	93	Grogan (1991)
	690	207	30	
	1,074	483	45	
	1,616	582	36	
	1,848	1,076	58	Siddharthan, Sebaaly, and Javaregowda (1992)[2]
	2,034	1,020	50	
	5,109	1,841	36	
	9,991	3,454	35	
Backcalculated moduli for base course, Mpa	96.2	21	22	Grogan (1991)
	110	47	42	Siddharthan, Sebaaly, and Javaregowda (1992)[2]
	128	66	51	
	207	27	13	Grogan (1991)
	242	244	101	
	401	180	45	

(Continued)

[1] Frequancy distributions appeared to be normal in shape.
[2] Frequency distributions appeared to have a positive skew.

Table E11 (Concluded)

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Backcalculated moduli for base course, Mpa	505	123	24	Siddharthan, Sebaaly, and Javaregowda (1992)[2]
	539	171	32	
Backcalculated moduli for subgrade, MPa	37	2.3	6.3	Siddharthan, Sebaaly, and Javaregowda (1992)[2]
	38	2.1	5.3	
	110	12	11	
	119	17	14	Grogan (1991)
	142	16	11	
	146	14	10	Siddharthan, Sebaaly, and Javaregowda (1992)[2]
	435	35	8.0	Grogan (1991)
	483	116	24	

[2] Frequency distributions appeared to have a positive skew.

Table E12
Initial Serviceability Index

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.2	0.33	7.9	Darter, Hudson, and Brown (1973)[1]

[1] Normality was not rejected by the chi-square "goodness-of-fit" test.

Appendix F
Portland Cement Concrete

Table F1
Air Content (Percent) of Concrete in its Plastic State

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.4	0.7	16	Hanna, McLaughlin, and Lott (1967)
4.6	0.8	17	Neamen and Laquros (1967)
4.6	0.8	18	Newlon (1966)
4.8	0.7	14	
4.8	0.6	13	Brown (1975)
5.0	0.8	16	Baker and McMahon (1969)
5.4	0.8	15	Willenbrock (1974b)[1]
5.4	1.0	19	SRCWV (1968)
5.5	0.8	15	Baker and McMahon (1969)
6.5	1.4	22	

[1] Frequency distribution appeared to have negative skew.
Note: On the average, sampling error and experimental error (testing and inherent) accounted for 25 percent of the total variance (Baker and McMahon 1969; Hanna, McLaughlin, and Lott 1967; SRCWV 1968; Neamen and Laquros 1967).

Table F2
Slump (mm) of Concrete in its Plastic State

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
38	8	20	Willenbrock (1979b)[1]
38	20	53	Neamen and Laquros (1967)
51	13	25	Kennedy, Hudson, and McCullough (1975)
51	15	30	Brown (1975)
53	15	28	Baker and McMahon (1969)
56	18	32	
56	18	31	Newlon (1966)
61	20	33	SRCWV (1968)
76	25	33	Newlon (1966)
76	25	33	Hanna, McLaughlin, and Lott (1967)

[1] Frequency distribution appeared to be normal in shape.
Note: On the average, sampling error and experimental error (testing and inherent) accounted for 20 percent of the total variance (Hanna, McLaughlin, and Lott 1967; SRCWV 1968; Neamen and Laquros 1967; Baker and McMahon 1969).

Table F3
Particle Size Distribution for Concrete Aggregates, Percent Finer

Sieve Size	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
25 mm	95.6	3.8	4.0	Louisiana Department of Highways (1966)
	96.4	1.6	1.7	Willenbrock (1974b)[1]
19 mm	69.1	11.5	17	Baker and McMahon (1969)
	71.5	12.6	18	
	75.4	10.8	14	Louisiana Department of Highways (1966)
	92.6	3.5	3.8	Baker and McMahon (1969)
13 mm	35.5	12.7	36	Louisiana Department of Highways (1966)
	39.9	6.7	17	Willenbrock (1974b)[1]
4.75 mm	1.3	1.2	92	Louisiana Department of Highways (1966)
	2.4	1.1	46	Willenbrock (1974b)[1]
2.36	1.4	0.9	64	Willenbrock (1974b)[1]
4.75 mm	96.2	1.1	1.1	Willenbrock (1974b)[1]
	97.8	1.5	1.5	Louisiana Department of Highways (1966)
2.36 mm	79.4	2.5	3.1	Willenbrock (1974b)[1]
1.18 mm	64.9	3.9	6.0	Willenbrock (1974b)[1]
0.60 mm	48.1	4.2	8.7	Willenbrock (1974b)[1]
0.30 mm	15.9	6.5	41	Louisiana Department of Highways (1966)
	20.6	2.7	13	Willenbrock (1974b)[1]
0.15 mm	2.1	1.3	62	Louisiana Department of Highways (1966)
	3.9	0.7	18	Willenbrock (1974b)[1]

[1] Frequency distribution appeared to be normal in shape.
Note: Sampling error and experimental error (testing and inherent) accounted for the following percentages of total variance.
- 25-mm sieve: 30 percent (Louisiana Department of Highways 1966).
- 19-mm sieve: 40 percent (Baker and McMahon 1969; LA Department of Highways 1966).
- 13-mm sieve: 85 percent (Louisiana Department of Highways 1966).
- 4.75-mm sieve: 50 percent (Louisiana Department of Highways 1966).

Note: Sampling error and experimental error (testing and inherent) accounted for the following percentages of total variance (Louisiana Department of Highways 1966).
- 4.75-mm sieve: 15 percent.
- 1.18-mm sieve: 8 percent.
- 0.15-mm sieve: 20 percent.

Table F4
Fineness Modulus of Concrete Aggregates

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
2.86	0.12	4.2	Willenbrock (1974b)[1]

[1] Frequency distribution appeared to be normal in shape.

Table F5
Density of Hardened Concrete, kg/m^3

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
2,371	14.3	0.6	Hanna, McLaughlin, and Lott (1967)
N/A	N/A	2.1 (1.3 to 2.4)	Kennedy, Hudson, and McCullough (1975)
N/A	N/A	1.7 (1.1 to 4.7)	Marshall and Kennedy (1974)

N/A Data not reported.

Table F6
Compressive Strength of Hardened Concrete, MPa

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
20	5.5	28	Highway Research Board (1962a)
26	5.0	19	Neamen and Laquros (1967)
26	2.0	7.6	Highway Research Board (1962a)
26	4.1	16	SRCWV (1966)
26	2.8	11	
27	4.9	18	State Road Depaartment of Florida (1965)
28	3.9	14	
28	4.2	15	
29	4.6	16	
29	3.8	13	Newlon (1966)
29	2.5	8.6	Highway Research Board (1962a)
31	3.5	12	Newlon (1966)
31	4.6	15	
32	3.9	12	SRCWV (1966)
32	5.5	17	
33	3.3	10	
33	2.7	8.0	Willenbrock (1974a)[1]
33	4.0	12	State Road Depaartment of Florida (1965)
34	4.7	14	
34	3.7	11	Highway Research Board (1962a)
35	4.2	12	State Road Depaartment of Florida (1965)
35	4.6	13	
40	2.8	6.9	Kennedy, Hudson, and McCullough (1975)
42	2.6	6.1	Highway Research Board (1962a)

[1] Frequency distribution appeared to be normal in shape.
Note: On the average, sampling error and experimental error (testing and inherent) accounted for 45 percent of the total variance (Neamen and Laquros 1967; Newlon 1966; SRCWV 1966).

Table F7
Compressive Modulus and Poisson's Ratio for Hardened Concrete

Property	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Compression Modulus, MPa	25,787	6,189	24	Marshall and Kennedy (1974)
	N/A	N/A	34 (21 to 49)	Kennedy, Hudson, and McCullough (1975)
Poisson's ratio	N/A	N/A	14 (9.4 to 20)	Gibeaut (1960)

N/A Data not reported.

Table F8
Measures of Tensile Stiffness and Strength

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Indirect tensile strength, kPa	3,248	648	20	Marshall and Kennedy (1974)
	3,806	269	7.0	Kennedy, Hudson, and McCullough (1975)[1]
	4,399	793	18	
Indirect tensile modulus, MPa	23,788	8,067	34	Kennedy, Hudson, and McCullough (1975)[1]
	27,511	9,377	34	Marshall and Kennedy (1974)
Flexural strength, kPa	3,792	255	6.7	Highway Research Board (1962a)
	4,344	241	5.6	
	4,461	359	8.0	Kennedy, Hudson, and McCullough (1975)
	4,564	434	9.5	Brown (1975)
	4,895	365	7.5	Highway Research Board (1962a)
	5,282	421	8.0	Brown (1975)
	6,068	365	6.0	Highway Research Board (1962a)

[1] Frequency distributions appeared to be normal in shape.
Note: Frequency distributions for flexural strength appeared normal in shape (Kher and Darter 1973).

Table F9
Slab Thickness, mm

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
211	7.6	3.6	Louisiana Department of Highways (1966)
211	6.1	2.9	Marshall and Kennedy (1974)
216	7.6	3.5	Oklahoma Department of Highways (1968)
226	2.5	1.1	Neamen and Laquros (1967)
226	2.5	1.1	Oklahoma Department of Highways (1968)
229	10	4.4	
234	7.1	3.0	Michigan Department of Highways (1966)[1]
234	7.4	3.2	Louisiana Department of Highways (1966)
241	11	4.7	Marshall and Kennedy (1974)
262	6.9	2.6	Louisiana Department of Highways (1966)
277	23	8.3	Kennedy, Hudson, and McCullough (1975)
376	12	3.3	

[1] Frequency distributions appeared to be normal in shape.

Table F10
Load Transfer (Percent)

Joint Type	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Doweled expansion	30.5	7.4	24	Rollings (1987)
Doweled construction	12.2[1]	3.6	29	Hammons, Pittman, and Mathews (1995)[3]
	15.5[1]	5.8	38	
	16.3[2]	4.1	25	
	19.3[1]	1.8	9.3	
	28.5[2]	4.6	16	
	30.6	11.6	38	Rollings (1987)
Keyed construction	25.4	10.5	41	Rollings (1987)
Doweled contraction	13.0[1]	6.6	50	Hammons, Pittman, and Mathews (1995)[3]
	16.3[1]	9.1	56	
	17.9[1]	3.9	22	
	21.2[1]	2.1	9.9	
	26.6[2]	3.1	12	
	30.7[2]	1.1	3.5	
Tied contraction	24.7[2]	8.6	35	Hammons, Pittman, and Mathews (1995)[4]
	25.5[1]	1.8	6.9	
	27.0[1]	4.3	16	
Plain contraction	6.4[1]	2.9	46	Hammons, Pittman, and Mathews (1995)[3]
	9.0[1]	7.4	82	
	10.6[2]	3.2	30	
	10.8[2]	3.2	30	
	11.9[1]	2.0	17	
	11.9[1]	3.4	29	
	12.8[2]	1.6	13	
	13.6[2]	1.6	12	
	14.2[2]	3.7	26	
	17.4[1]	6.6	38	

(Continued)

[1] Cold testing conditions.
[2] Warm testing conditions.
[3] Frequency distribution appeared to have positive skew.
[4] Frequency distribution appeared to be normal in shape.

Table F10 (Concluded)

Joint Type	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Plain contraction (Cont.)	17.4[2]	3.4	19	Hammons, Pittman, and Mathews (1995)[3]
	17.5[1]	4.3	25	
	17.6[2]	2.2	13	
	17.6[1]	5.3	30	
	20.0[2]	3.1	16	
	20.3[2]	5.5	27	
	20.4[1]	6.7	33	
	20.8[1]	2.6	12	
	25.2[2]	2.2	8.7	
	37.2	7.1	19	Rollings (1987)

[1] Cold testing conditions.
[2] Warm testing conditions.
[3] Frequency distribution appeared to have positive skew.

Table F11
Falling Weight Deflectometer Results

Measure	Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
Deflections under load, μm	81	8.9	11	Grogan (1991)
	102	2.9	2.8	
	396	45	11	
	4,53	121	27	
Backcalculated moduli for portland cement concrete, MPa	25,690	2,830	11	
	29,150	15,740	54	
Backcalculated moduli for subgrade, MPa	80	14	17	
	109	17	16	
	167	32	19	

Table F12
Serviceability Index for Rigid Pavements

Mean	Standard Deviation	Coefficient of Variation, Percent	Reference
4.7	0.14	3.0	Fordyce and Teske (1963)

[1] Frequency distribution appeared to be normal in shape.

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This report provides a summary of published data concerning the variability of pavement materials and structures. In addition, related statistical concepts and data analysis techniques are reviewed. This study was initiated as the first step toward the development of statistical specifications for pavement construction.

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